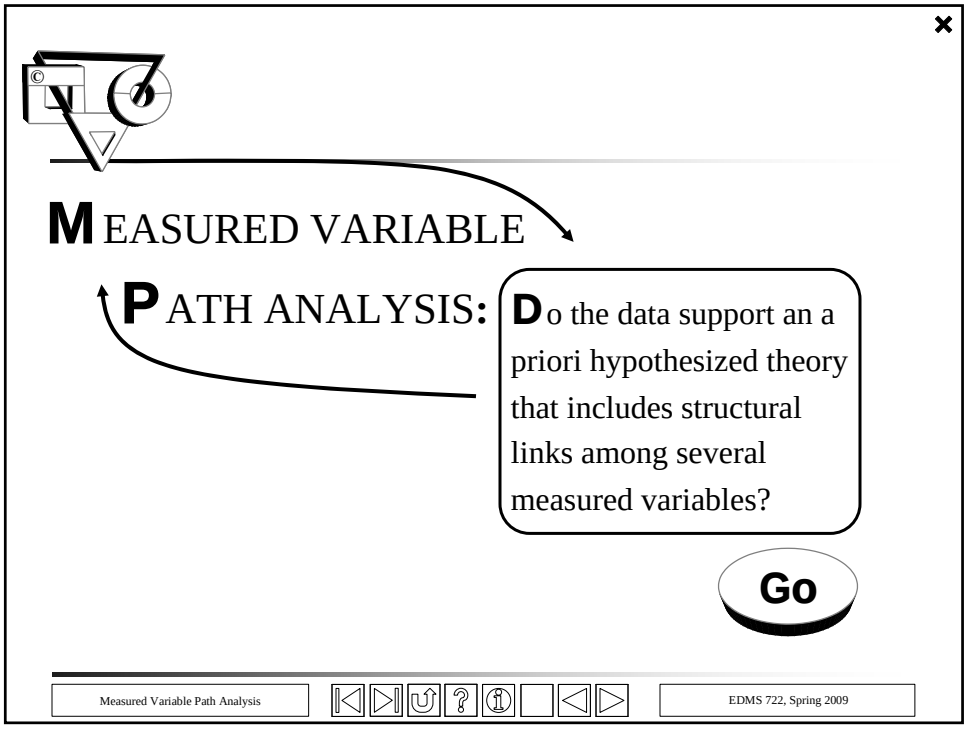
MEASURED VARIABLE
PATH ANALYSIS:
Do the data support an a priori hypothesized theory that includes structural links among several measured variables?
Go
Measured Variable Path Analysis
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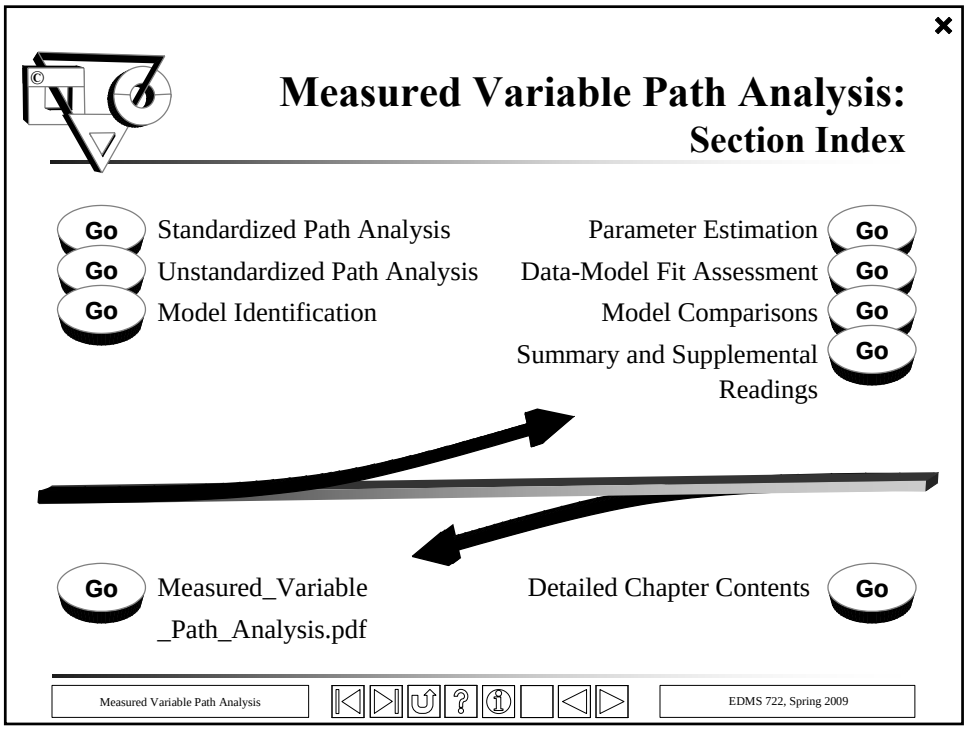
Measured Variable Path Analysis:
Section Index
Go Standardized Path Analysis
Go Unstandardized Path Analysis
Go Model Identification
Parameter Estimation Go
Data-Model Fit Assessment Go
Model Comparisons Go
Summary and Supplemental Readings Go
Go Measured_Variable_Path_Analysis.pdf
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STANDARDIZED

PATH

ANALYSIS

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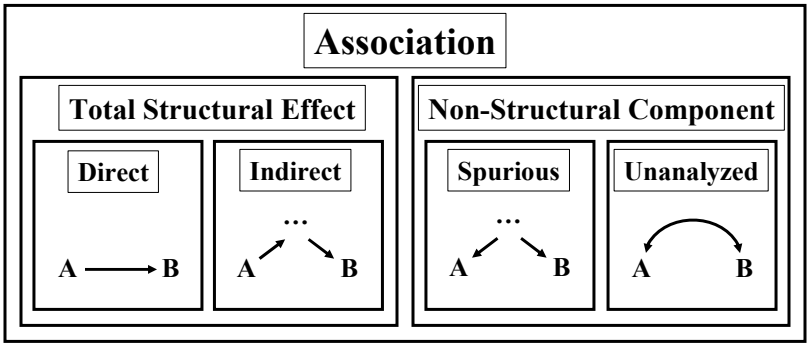
The Decomposition of Correlations

- **In standardized path analysis correlations are decomposed into structural (i.e., causal) and nonstructural (i.e., non-causal) components, based on the theory expressed in the path diagram.**



Measured Variable Path Analysis

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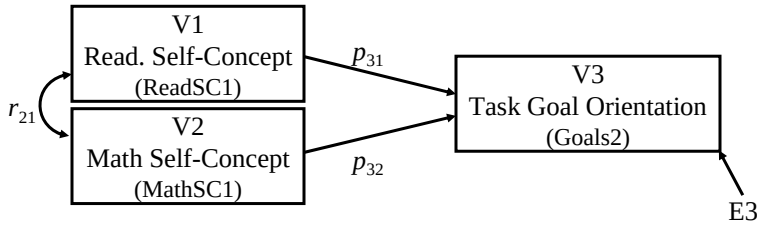
Multiple Linear Regression: The Structural Equation

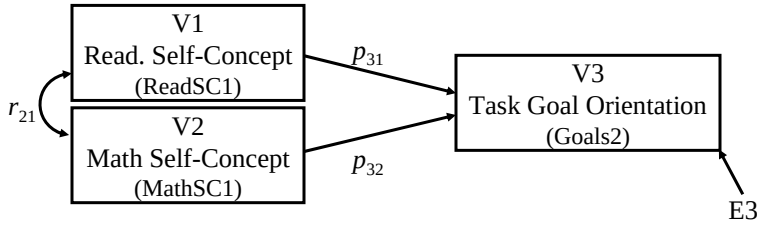


- **Structural Equation (variables are standardized):**

$$V3 = p_{31}V1 + p_{32}V2 + E3$$

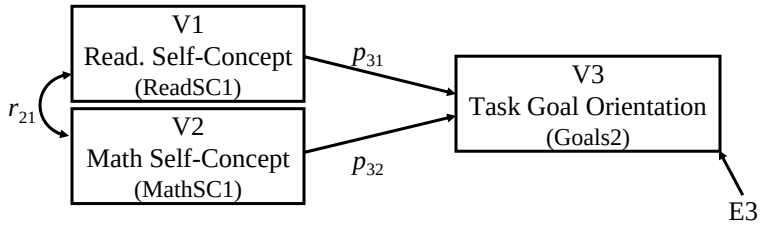
A structural equation is a regression-type equation expressing each endogenous variable as a function of all elements having a *direct* structural effect on it.

Multiple Linear Regression: The Decomposition of Correlations



	total struc. effect (TE)		non-struc. component		model-implied correlation
	Direct (DE)	Indirect (IE)	Spurious	Unanalyzed	
V1,V3	p_{31}	--	--	$r_{21}p_{32}$	$p_{31}+r_{21}p_{32}$
V2,V3	p_{32}	--	--	$r_{21}p_{31}$	$p_{32}+r_{21}p_{31}$
V1,V2	--	--	--	r_{21}	r_{21}

Multiple Linear Regression: Model-Implied vs. Observed Correlations



Model-Implied Correlations

	V1	V2	V3
V1	1		
V2	r_{21}	1	
V3	$p_{31}+r_{21}p_{32}$	$p_{32}+r_{21}p_{31}$	1

=

Sample Correlations

	V1	V2	V3
V1	1		
V2	-.173	1	
V3	.122	.163	1

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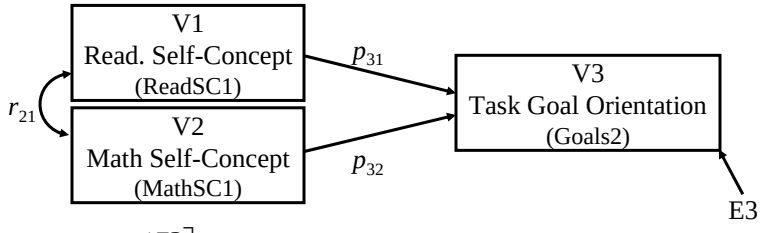
Multiple Linear Regression: Parameter Estimation



$$\begin{bmatrix} r_{21} = -.173 \\ p_{31} + r_{21}p_{32} = .122 \\ p_{32} + r_{21}p_{31} = .163 \end{bmatrix} \Rightarrow \begin{bmatrix} p_{31} - .173p_{32} = .122 \\ p_{32} - .173p_{31} = .163 \end{bmatrix}$$

$$\begin{bmatrix} p_{31} = .122 + .173p_{32} \\ p_{32} - .173(.122 + .173p_{32}) = .163 \end{bmatrix} \Rightarrow \begin{bmatrix} \hat{r}_{21} = -.173 \\ \hat{p}_{31} = .155 \\ \hat{p}_{32} = .190 \end{bmatrix}$$

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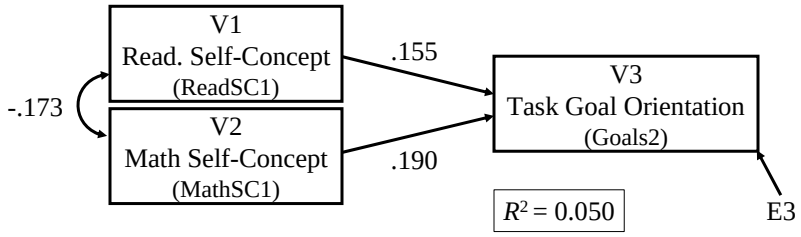
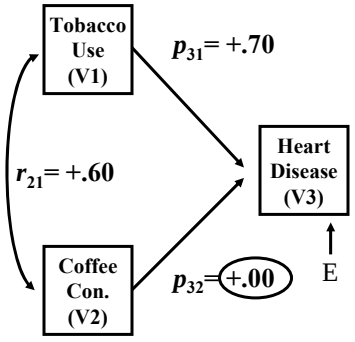
Multiple Linear Regression: Path Analysis Interpretation



- Assuming a correct underlying model/theory:
  - A one standard deviation (SD) increase in V1 causes, on average, a .155 SD increase in V3, holding all else constant.
  - A one SD increase in V2 causes, on average, a .190 SD increase in V3, holding all else constant.

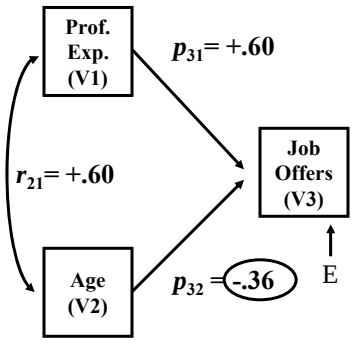
Sidebar: Correlations vs. Direct Effects - I

$$\mathbf{P} = \begin{array}{c} \begin{matrix} \text{Tobacco Use (V1)} & \text{Coffee Consumption (V2)} & \text{Heart Disease (V3)} \end{matrix} \\ \begin{bmatrix} 1 & & \\ +.60 & 1 & \\ +.70 & +.42 & 1 \end{bmatrix} \end{array}$$



Sidebar: Correlations vs. Direct Effects - II

	Professional Experience (V1)	Age (V2)	No. of Job Offers (V3)
P =	1		
	+.60	1	
	+.38	(+.00)	1



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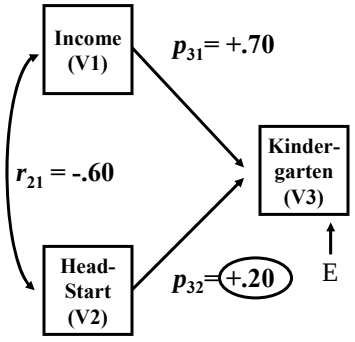
Sidebar: Correlations vs. Direct Effects - III

	Income (V1)	Head-Start (V2)	Kinder-garten (V3)
P =	1		
	−.60	1	
	+.58	(−.22)	1



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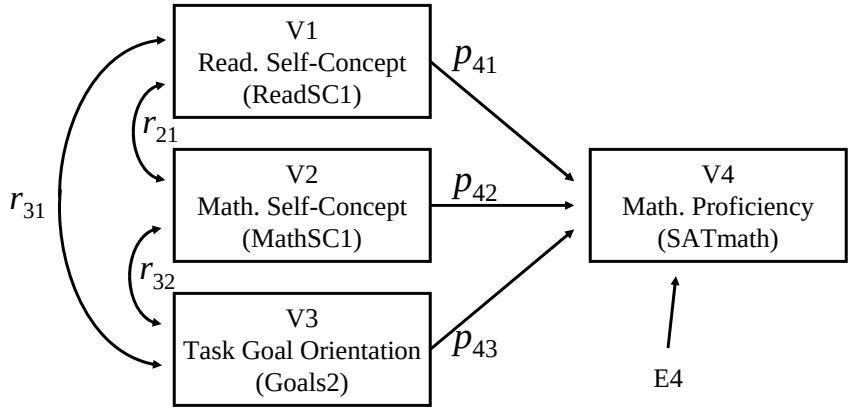
Sidebar:
Correlations vs. Direct Effects

"If … we choose a group of social phenomena with no antecedent knowledge of the causation or the absence of causation among them, then calculation of correlation coefficients, total or partial, will not advance us a step towards evaluating the importance of the causes at work."

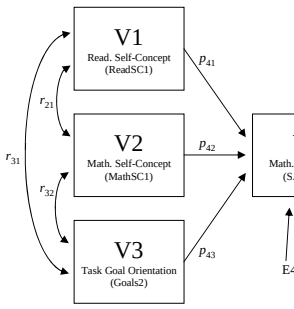
R. A. Fisher, 1958 (p. 190)

Standardized Path Analysis:
A Second MLR Example



$$V4 = p_{41}V1 + p_{42}V2 + p_{43}V3 + E4$$

A Second MLR Example:
Model-Implied vs. Observed Correlations



	structural		non-structural	model-implied	obs.
	DE	IE			
V4,V1	p_{41}	--	$r_{21}p_{42}+r_{31}p_{43}$	$p_{41}+r_{21}p_{42}+r_{31}p_{43}$	-.049
V4,V2	p_{42}	--	$r_{21}p_{41}+r_{32}p_{43}$	$p_{42}+r_{21}p_{41}+r_{32}p_{43}$	.556
V4,V3	p_{43}	--	$r_{31}p_{41}+r_{32}p_{42}$	$p_{43}+r_{31}p_{41}+r_{32}p_{42}$	.385
V2,V1	--	--	r_{21}	r_{21}	-.173
V3,V1	--	--	r_{31}	r_{31}	.122
V3,V2	--	--	r_{32}	r_{32}	.163

A Second MLR Example:
Parameter Estimation

$$p_{41} + r_{21}p_{42} + r_{31}p_{43} = -.049$$

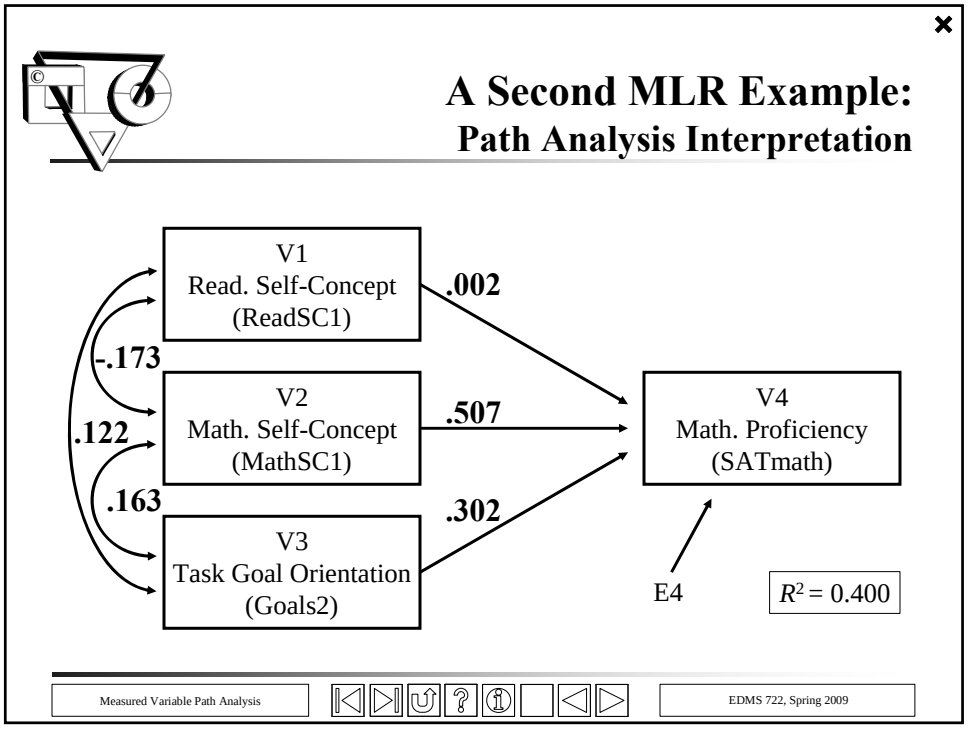
$$p_{42} + r_{21}p_{41} + r_{32}p_{43} = .556$$

$$p_{43} + r_{31}p_{41} + r_{32}p_{42} = .385$$

$$r_{21} = -.173$$

$$r_{31} = .122$$

$$r_{32} = .163$$


A Second MLR Example:
Path Analysis Interpretation
V1
Read. Self-Concept
(ReadSC1)
V2
Math. Self-Concept
(MathSC1)
V3
Task Goal Orientation
(Goals2)
V4
Math. Proficiency
(SATmath)
E4
.002
.507
.302
-.173
.122
.163
R2 = 0.400
Measured Variable Path Analysis
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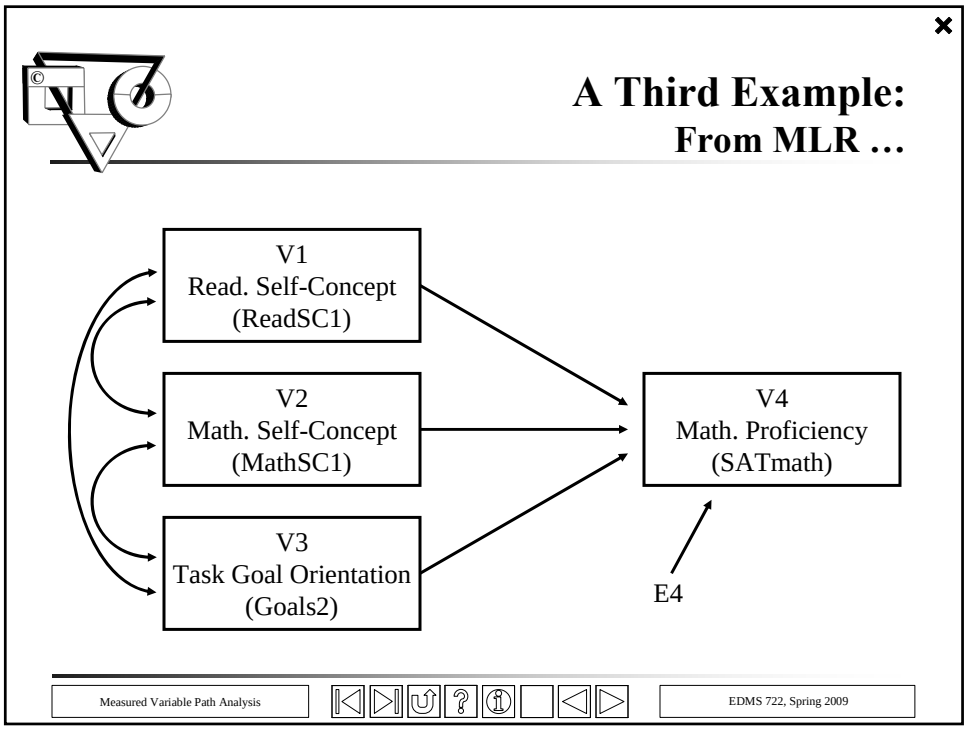
A Third Example:
From MLR …
V1
Read. Self-Concept
(ReadSC1)
V2
Math. Self-Concept
(MathSC1)
V3
Task Goal Orientation
(Goals2)
V4
Math. Proficiency
(SATmath)
E4
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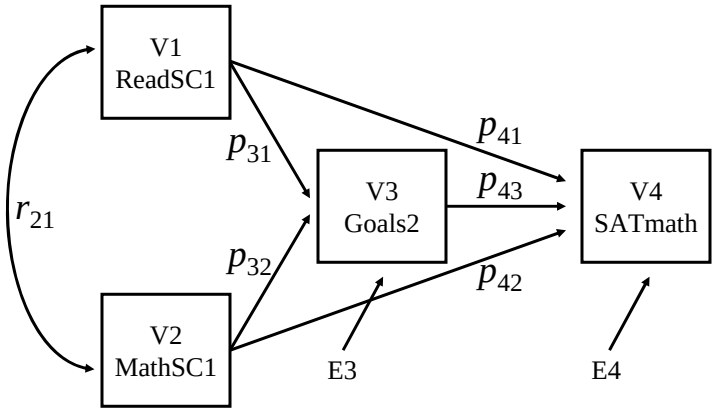
A Third Example:
… to Path Analysis



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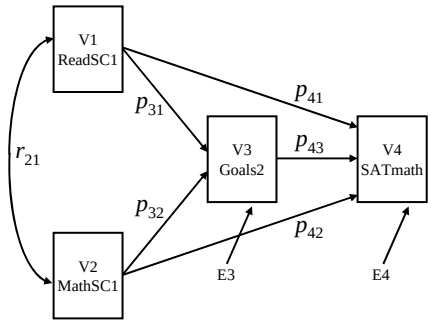
A Third Example:
The Structural Equations



$$V3 = p_{31}V1 + p_{32}V2 + E3$$

$$V4 = p_{41}V1 + p_{42}V2 + p_{43}V3 + E4$$

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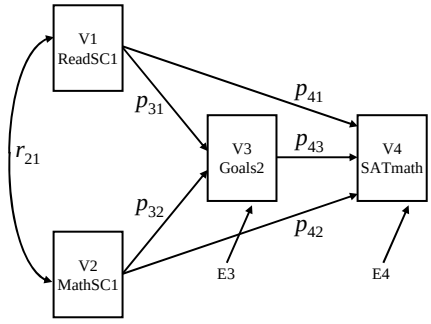
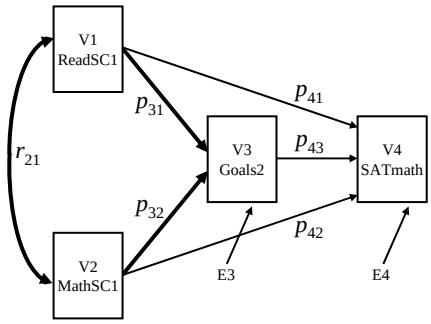
Formalizing Correlation Decomposition



A model implies sources of correlation between any two variables. These *traces* are products of model parameters and may be heuristically derived as follows:

- **Standardized Path Tracing Rules:**
  - One can go forward or backward causally; but once gone forward, one cannot go backward.
  - One can go through only one unanalyzed relation (two-headed arrow).
  - One can enter or leave a variable only once.

A Third Example: Path Tracing



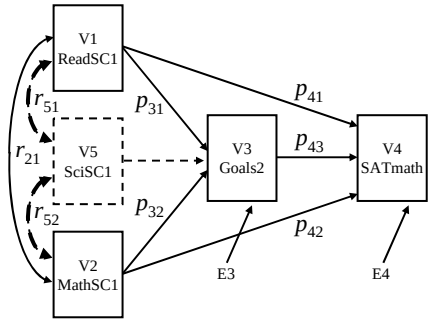
Following the Path Tracing Rules:

- One can go forward or backward causally; but once gone forward, one cannot go backward.
- One can go through only one unanalyzed relation (two-headed arrow).
- One can enter or leave a variable only once.

V2, V1: r_{21} **(unanalyzed)**

but not: ~~$p_{31}p_{32}$~~

A Third Example: Path Tracing



Following the Path Tracing Rules:

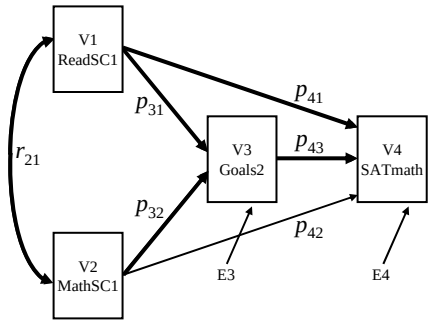
- One can go forward or backward causally; but once gone forward, one cannot go backward.
- One can go through only one unanalyzed relation (two-headed arrow).
- One can enter or leave a variable only once.

V2, V1: r_{21} **(unanalyzed)**

but not: ~~$r_{21}r_{52}$~~

(if V5 were a 3rd exogenous variable)

A Third Example: Path Tracing



Following the Path Tracing Rules:

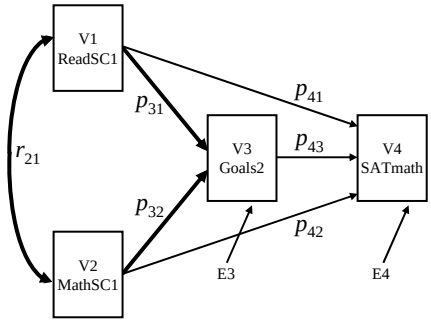
- One can go forward or backward causally; but once gone forward, one cannot go backward.
- One can go through only one unanalyzed relation (two-headed arrow).
- One can enter or leave a variable only once.

V3, V1: p_{31} **(direct)**

$r_{21}p_{32}$ **(unanalyzed)**

but not: ~~$p_{41}p_{43}$~~

A Third Example: Path Tracing



Following the Path Tracing Rules:

- One can go forward or backward causally; but once gone forward, one cannot go backward.
- One can go through only one unanalyzed relation (two-headed arrow).
- One can enter or leave a variable only once.

V3, V1:	p_{31}	**(direct)**
	$r_{21}p_{32}$	**(unanalyzed)**
V3, V2:	p_{32}	**(direct)**
	$r_{21}p_{31}$	**(unanalyzed)**

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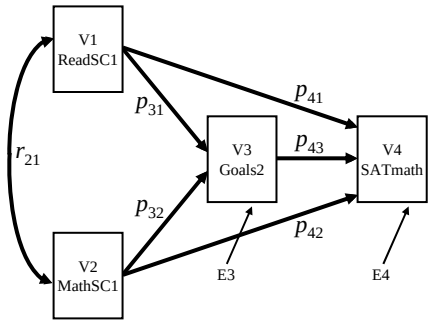
A Third Example: Path Tracing



Following the Path Tracing Rules:

- One can go forward or backward causally; but once gone forward, one cannot go backward.
- One can go through only one unanalyzed relation (two-headed arrow).
- One can enter or leave a variable only once.

V4, V1:	p_{41}	**(direct)**
	$p_{31}p_{43}$	**(indirect)**
	$r_{21}p_{32}p_{43}$	**(unanalyzed)**
	$r_{21}p_{42}$	**(unanalyzed)**

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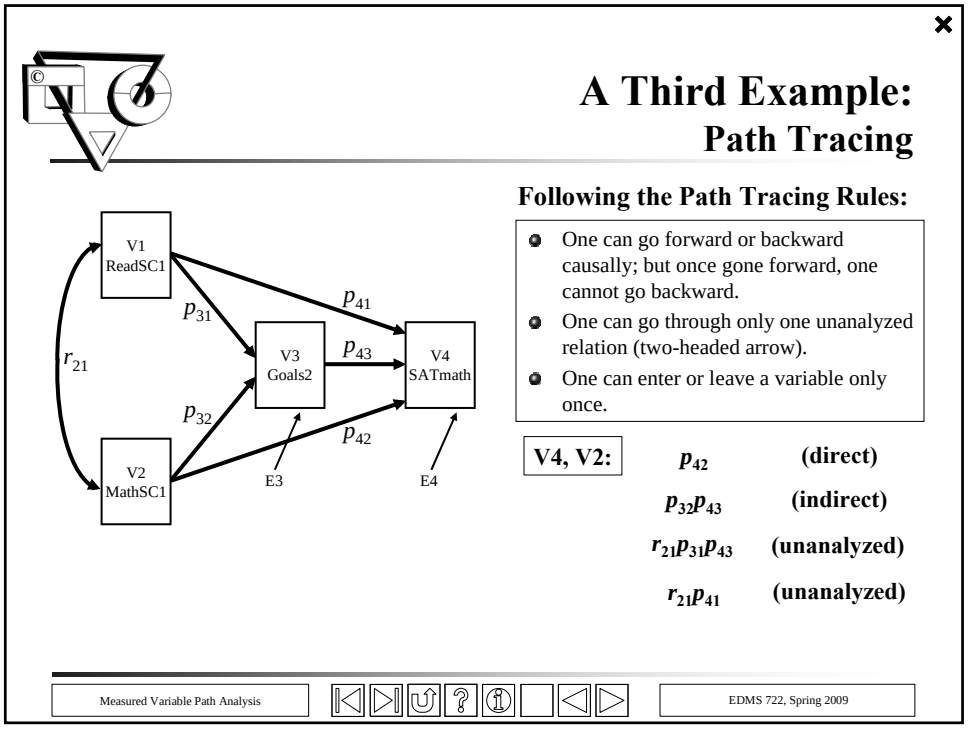
A Third Example:
Path Tracing
Following the Path Tracing Rules:
One can go forward or backward causally; but once gone forward, one cannot go backward.
One can go through only one unanalyzed relation (two-headed arrow).
One can enter or leave a variable only once.
V1 ReadSC1
V2 MathSC1
V3 Goals2
V4 SATmath
E3
E4
p_{31}
p_{41}
p_{43}
p_{32}
p_{42}
r_{21}
V4, V2:
p_{42} (direct)
$p_{32}p_{43}$ (indirect)
$r_{21}p_{31}p_{43}$ (unanalyzed)
$r_{21}p_{41}$ (unanalyzed)
Measured Variable Path Analysis
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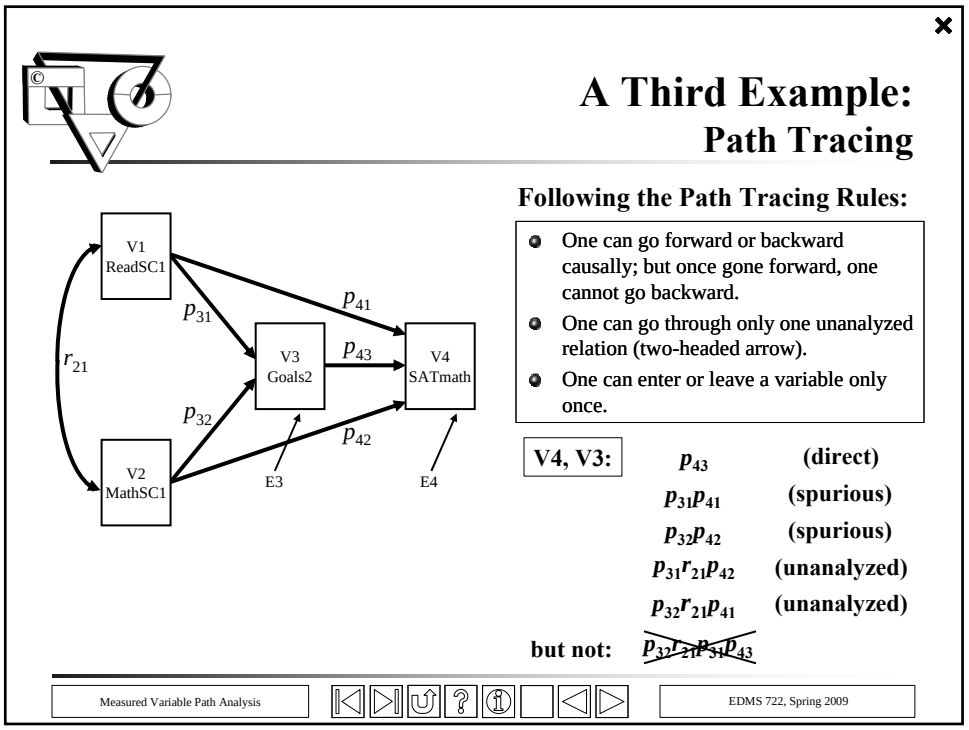
A Third Example:
Path Tracing
Following the Path Tracing Rules:
One can go forward or backward causally; but once gone forward, one cannot go backward.
One can go through only one unanalyzed relation (two-headed arrow).
One can enter or leave a variable only once.
V1 ReadSC1
V2 MathSC1
V3 Goals2
V4 SATmath
E3
E4
p_{31}
p_{41}
p_{43}
p_{32}
p_{42}
r_{21}
V4, V3:
p_{43} (direct)
$p_{31}p_{41}$ (spurious)
$p_{32}p_{42}$ (spurious)
$p_{31}r_{21}p_{42}$ (unanalyzed)
$p_{32}r_{21}p_{41}$ (unanalyzed)
but not: $p_{32}r_{21}p_{31}p_{43}$
Measured Variable Path Analysis
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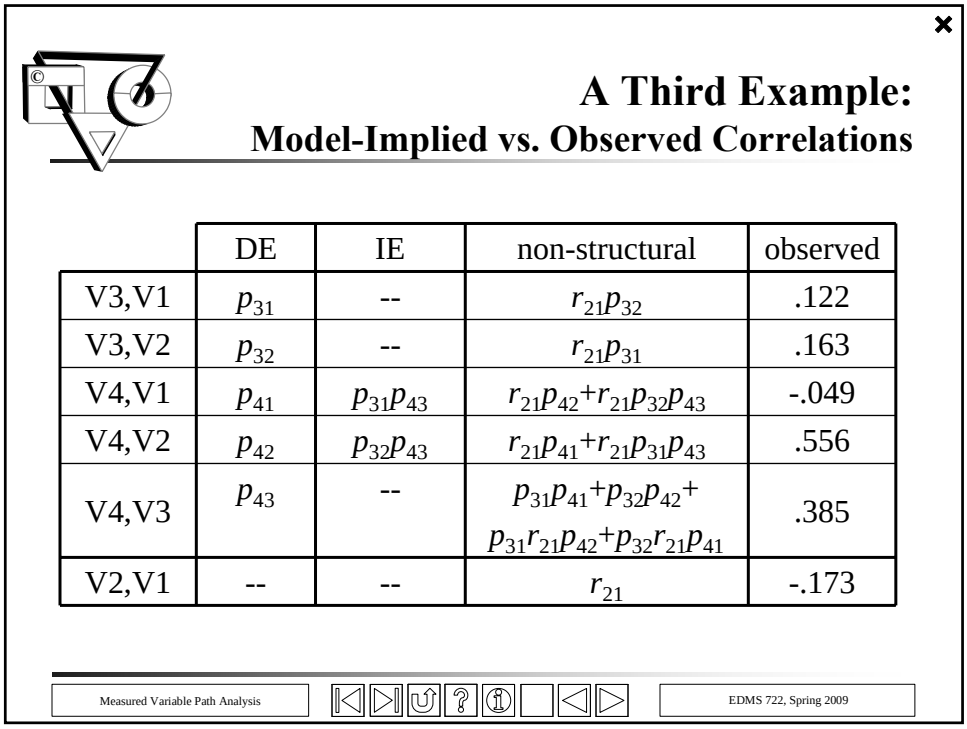

A Third Example:
Model-Implied vs. Observed Correlations

	DE	IE	non-structural	observed
V3,V1	p_{31}	--	$r_{21}p_{32}$	.122
V3,V2	p_{32}	--	$r_{21}p_{31}$	.163
V4,V1	p_{41}	$p_{31}p_{43}$	$r_{21}p_{42}+r_{21}p_{32}p_{43}$	-.049
V4,V2	p_{42}	$p_{32}p_{43}$	$r_{21}p_{41}+r_{21}p_{31}p_{43}$	.556
V4,V3	p_{43}	--	$p_{31}p_{41}+p_{32}p_{42}+$ $p_{31}r_{21}p_{42}+p_{32}r_{21}p_{41}$	.385
V2,V1	--	--	r_{21}	-.173

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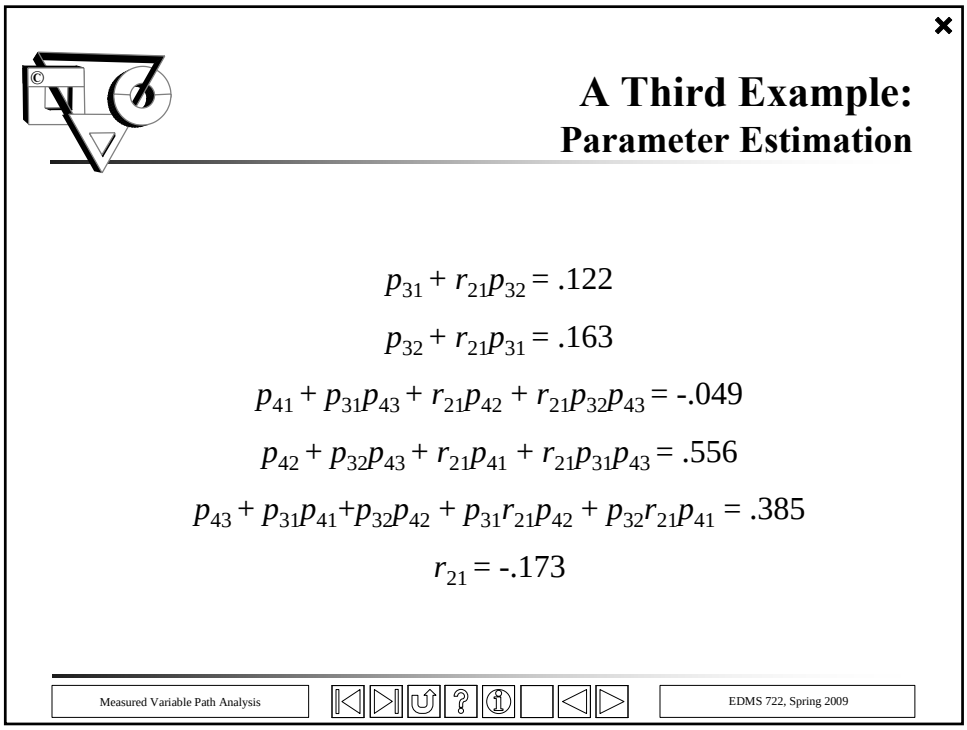

A Third Example:
Parameter Estimation

$$p_{31} + r_{21}p_{32} = .122$$

$$p_{32} + r_{21}p_{31} = .163$$

$$p_{41} + p_{31}p_{43} + r_{21}p_{42} + r_{21}p_{32}p_{43} = -.049$$

$$p_{42} + p_{32}p_{43} + r_{21}p_{41} + r_{21}p_{31}p_{43} = .556$$

$$p_{43} + p_{31}p_{41}+p_{32}p_{42} + p_{31}r_{21}p_{42} + p_{32}r_{21}p_{41} = .385$$

$$r_{21} = -.173$$

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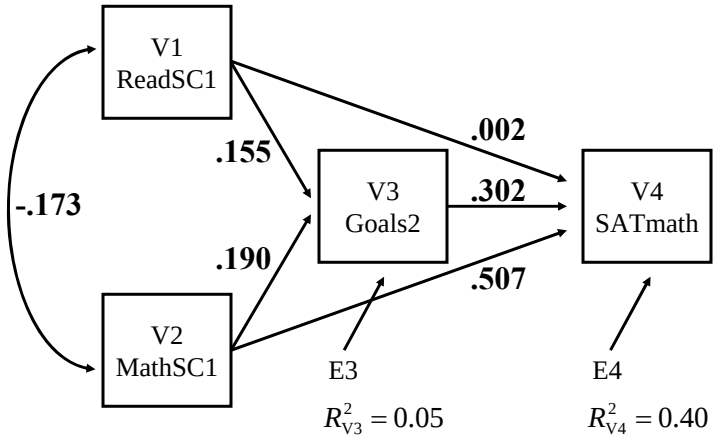
A Third Example: Interpretation



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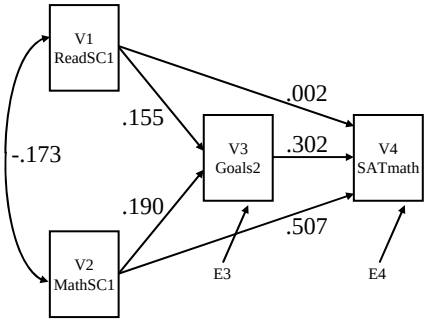
A Third Example: Decomposition of Structural Effects



	DE [+] IE [=] TE			obs. corr.
Goals2, ReadSC1	.155	--	.155	.122
Goals2, MathSC1	.190	--	.190	.163
SATmath, ReadSC1	.002	.047	.049	-.049
SATmath, MathSC1	.507	.057	.564	.556
SATmath, Goals2	.302	--	.302	.385

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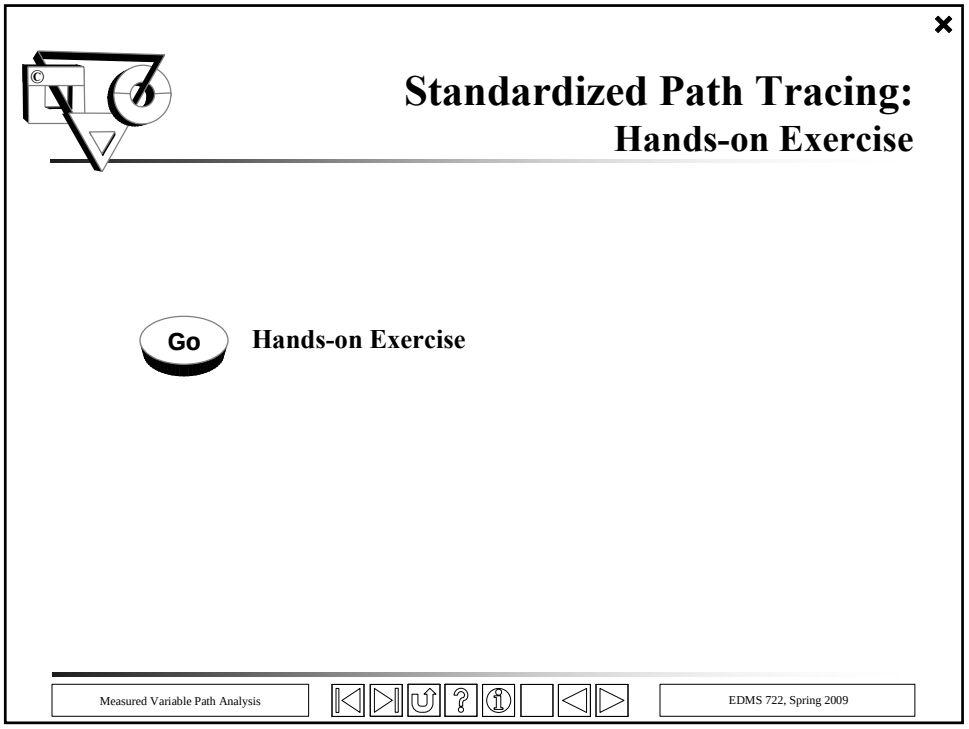
Standardized Path Tracing:
Hands-on Exercise
Go
Hands-on Exercise
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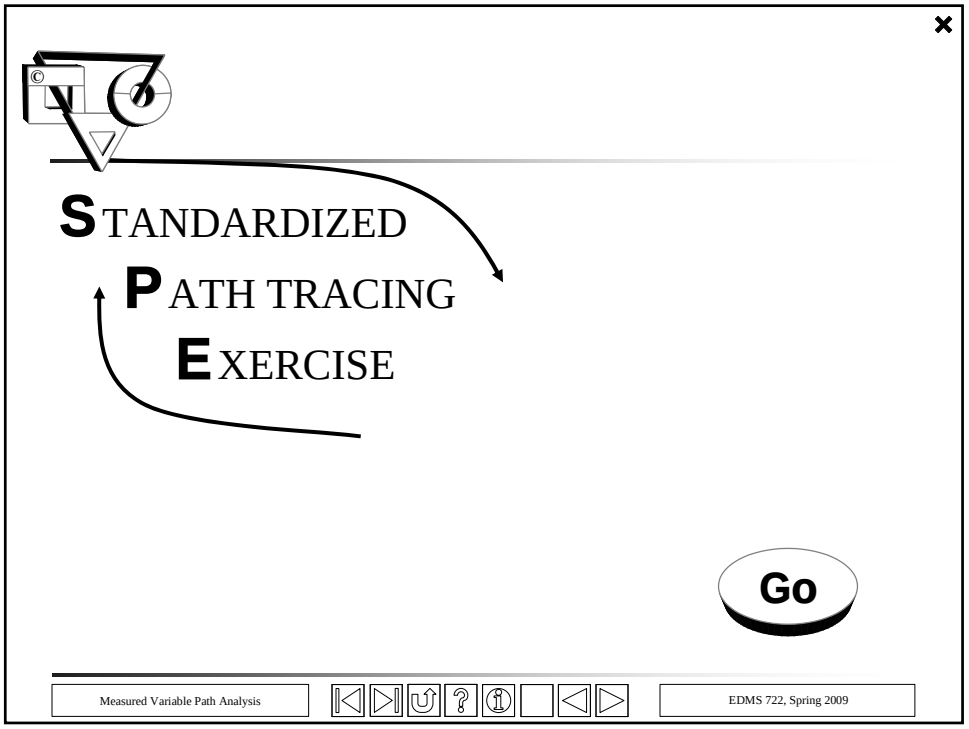
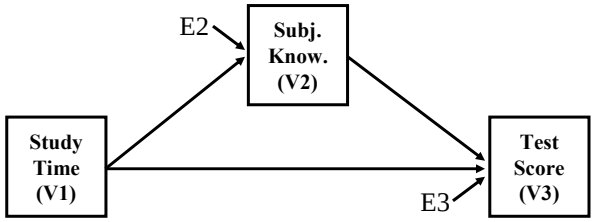
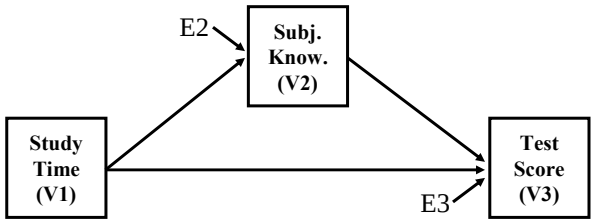

STANDARDIZED
PATH TRACING
EXERCISE
Go
Measured Variable Path Analysis
EDMS 722, Spring 2009

Basics of the Model



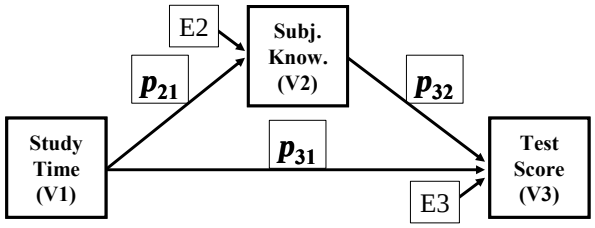
- List all endogenous elements.
- List all exogenous elements.
- Label standardized paths among measured variables.
- Write out all structural equations.
- Complete the table on the next slide.

Standardized Effect Decomposition



	direct effect?	indirect effect?	spurious?	un-analyzed?	model-implied?
V2, V1					
V3, V1					
V3, V2					

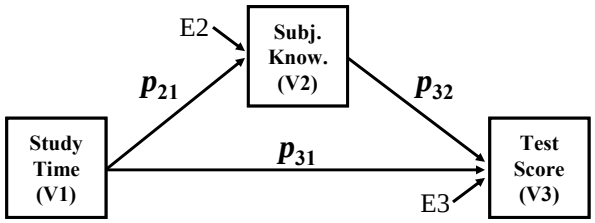
Exercise Solutions:
Basics of the Model



- List all endogenous elements: **V2 V3**
- List all exogenous elements: **V1 E2 E3**
- Label standardized paths among measured variables.
- Write out all structural equations: $\mathbf{V2 = p_{21}V1 + E2}$

$$\mathbf{V3 = p_{31}V1 + p_{32}V2 + E3}$$

Exercise Solutions:
Standardized Effect Decomposition



	direct effect?	indirect effect?	spurious?	un-analyzed?	model-implied?
V2, V1	p_{21}	--	--	--	p_{21}
V3, V1	p_{31}	$p_{32}p_{21}$	--	--	$p_{31}+p_{32}p_{21}$
V3, V2	p_{32}	--	$p_{31}p_{21}$	--	$p_{32}+p_{31}p_{21}$

Exercise Solutions:
Solving for Parameter Estimates

Model-Implied Correlations

	V1	V2	V3
V1	1		
V2	p_{21}	1	
V3	$p_{31}+p_{32}p_{21}$	$p_{32}+p_{31}p_{21}$	1

=

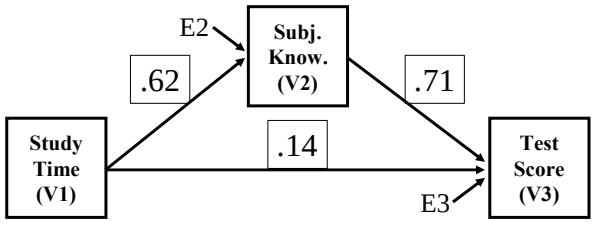
Sample Correlations

	V1	V2	V3
V1	1		
V2	.62	1	
V3	.58	.80	1

$$\begin{bmatrix} p_{21} = .62 \\ p_{31} + p_{32}p_{21} = .58 \\ p_{32} + p_{31}p_{21} = .80 \end{bmatrix} \Rightarrow \begin{bmatrix} p_{31} + .62p_{32} = .58 \\ p_{32} + .62p_{31} = .80 \end{bmatrix}$$

$$\begin{bmatrix} p_{31} = .58 - .62p_{32} \\ p_{32} + .62(.58 - .62p_{32}) = .80 \end{bmatrix} \Rightarrow \begin{bmatrix} \hat{p}_{21} = .62 \\ \hat{p}_{31} = .14 \\ \hat{p}_{32} = .71 \end{bmatrix}$$

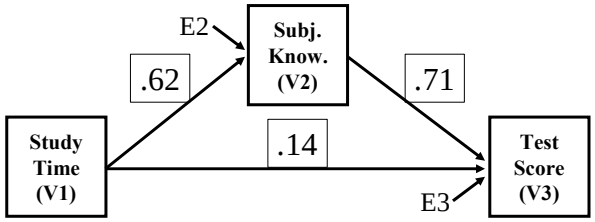
Exercise Solutions:
Standardized Parameter Estimates



$$\begin{bmatrix} p_{21} = .62 \\ p_{31} + p_{32}p_{21} = .58 \\ p_{32} + p_{31}p_{21} = .80 \end{bmatrix} \Rightarrow \begin{bmatrix} p_{31} + .62p_{32} = .58 \\ p_{32} + .62p_{31} = .80 \end{bmatrix}$$

$$\begin{bmatrix} p_{31} = .58 - .62p_{32} \\ p_{32} + .62(.58 - .62p_{32}) = .80 \end{bmatrix} \Rightarrow \begin{bmatrix} \hat{p}_{21} = .62 \\ \hat{p}_{31} = .14 \\ \hat{p}_{32} = .71 \end{bmatrix}$$

Exercise Solutions:
Standardized Parameter Estimates



	direct effect?	indirect effect?	spurious?	un-analyzed?	model-implied?
V2, V1	.62	--	--	--	.62
V3, V1	.14	.44	--	--	.58
V3, V2	.71	--	.09	--	.80

Measured Variable Path Analysis EDMS 722, Spring 2009

STANDARDIZED
PATH TRACING
EXERCISE

End of Section. Exit: Go

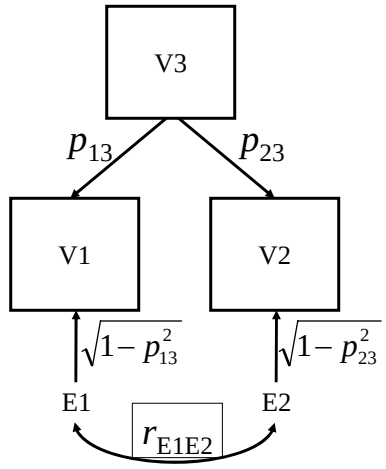
Measured Variable Path Analysis EDMS 722, Spring 2009

Path Tracing Sidebar: The familiar partial correlation formula

Partial correlation is the correlation between two variables (say, V1 and V2) after controlling for one or more other variables (say, V3).

$$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{1-r_{13}^2}\sqrt{1-r_{23}^2}}$$

Partial correlation is often explained as the correlation between two residualized variables (say, the residual of V1 and V2 after predicting each from V3).

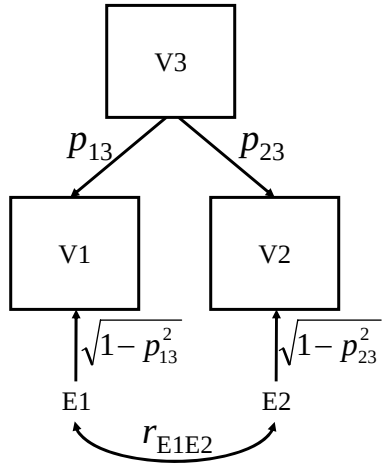


Path Tracing Sidebar: The familiar partial correlation formula (cont.)

$$r_{12} = \underbrace{p_{13}p_{23}}_{\text{“spurious”}} + \underbrace{\left(\sqrt{1-p_{13}^2}\right)r_{E1E2}\left(\sqrt{1-p_{23}^2}\right)}_{\text{“unanalyzed”}}$$

$$r_{12} - p_{13}p_{23} = \left(\sqrt{1-p_{13}^2}\right)r_{E1E2}\left(\sqrt{1-p_{23}^2}\right)$$

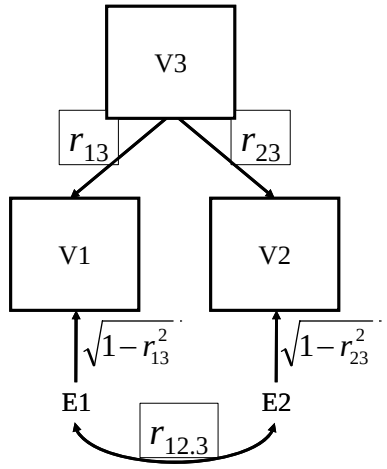
$$r_{E1E2} = \frac{r_{12} - p_{13}p_{23}}{\sqrt{1-p_{13}^2}\sqrt{1-p_{23}^2}}$$



Path Tracing Sidebar:
The familiar partial correlation formula (cont.)

$$r_{E1E2} = \frac{r_{12} - p_{13}p_{23}}{\sqrt{1-p_{13}^2}\sqrt{1-p_{23}^2}}$$

$$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{1-r_{13}^2}\sqrt{1-r_{23}^2}}$$

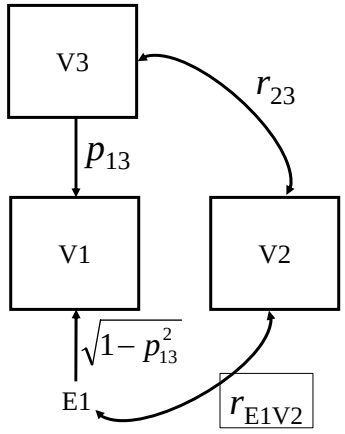


Path Tracing Sidebar:
The semi-partial correlation formula

Semi-partial (or *part*) correlation is the correlation between two variables (say, V1 and V2) after controlling for one or more other variables (say, V3) from just one of those variables (say, V1).

$$r_{2(1.3)} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{1-r_{13}^2}}$$

Semi-partial correlation is often explained as the correlation between one residualized variable (say, the residual of V1 after predicting it from V3) with one original variable.

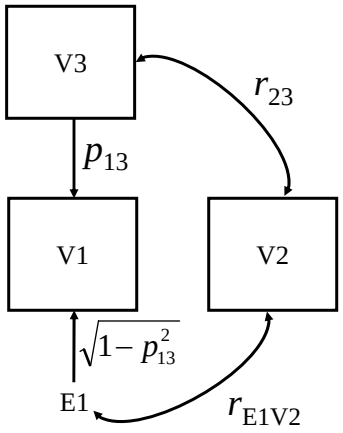


Path Tracing Sidebar:
The semi-partial correlation formula (cont.)

$$r_{12} = p_{13}r_{23} + \left(\sqrt{1-p_{13}^2}\right)r_{E1V2}$$

$$r_{12} - p_{13}r_{23} = \left(\sqrt{1-p_{13}^2}\right)r_{E1V2}$$

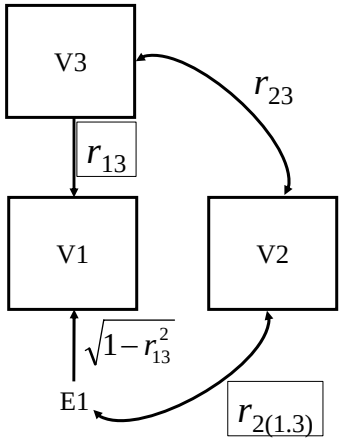
$$r_{E1V2} = \frac{r_{12} - p_{13}r_{23}}{\sqrt{1-p_{13}^2}}$$



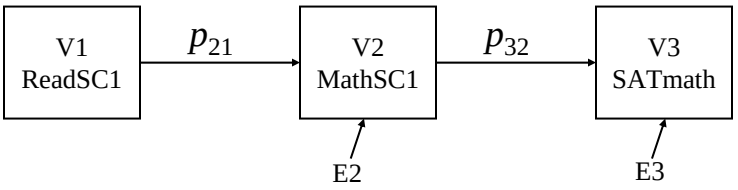
Path Tracing Sidebar:
The semi-partial correlation formula (cont.)

$$r_{E1V2} = \frac{r_{12} - p_{13}r_{23}}{\sqrt{1-p_{13}^2}}$$

$$r_{2(1.3)} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{1-r_{13}^2}}$$



A Fourth Example: The Structural Equations

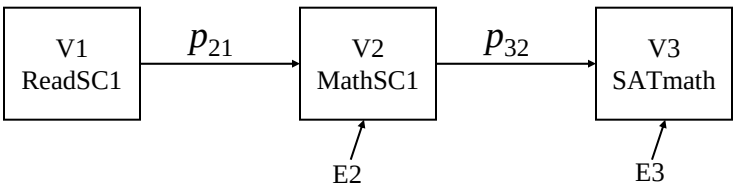


- **Structural Equations (variables are standardized):**

$V2 = p_{21}V1 + E2$

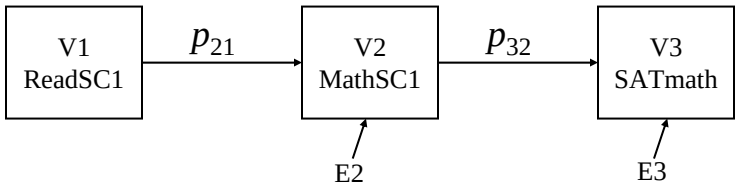
$V3 = p_{32}V2 + E3$

A Fourth Example: The Decomposition of Correlations



	DE (+)	IE (=)	TE	non-structural	model-implied
V2, V1	p_{21}	--	p_{21}	--	p_{21}
V3, V1	--	$p_{32}p_{21}$	$p_{32}p_{21}$	--	$p_{32}p_{21}$
V3, V2	p_{32}	--	p_{32}	--	p_{32}

A Fourth Example:
Model-Implied vs. Observed Correlations



Model-Implied Correlations

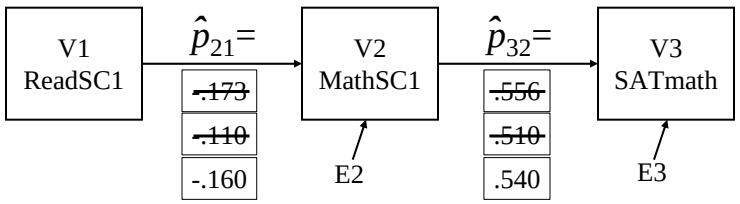
	V1	V2	V3
V1	1		
V2	p_{21}	1	
V3	$p_{32}p_{21}$	p_{32}	1

=

Sample Correlations

	V1	V2	V3
V1	1		
V2	-.173	1	
V3	-.049	.556	1

A Fourth Example:
Parameter Estimation?



$$\hat{p}_{32}\hat{p}_{21} = \begin{array}{|l} \sout{-.096} \neq -.049 \\ \sout{-.056} \neq -.049 \\ -.086 \neq -.049 \end{array}$$

$$\hat{p}_{21} = ? \approx -.173$$
$$\hat{p}_{32} = ? \approx +.556$$
$$\rightarrow \quad \hat{p}_{32}\hat{p}_{21} \approx -.049$$

Unstandardized path analysis will facilitate solving this and more complex estimation problems.