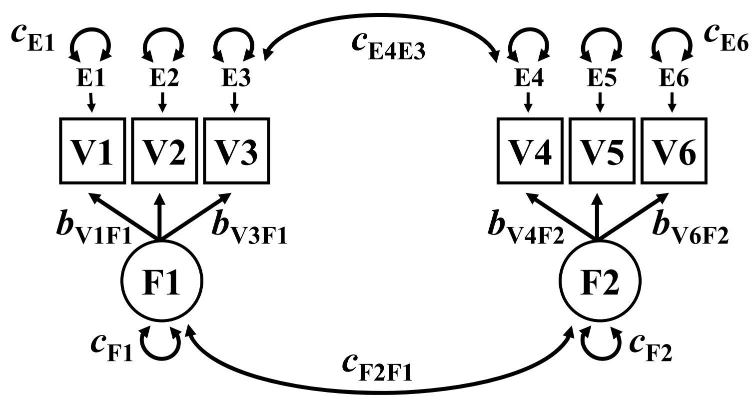
CFA REVIEW

Do the data support an a priori theory that includes structural links from latent factors to measured variables?

Go

a-b-c Notation



In general:
$b_{\text{to, from}}$

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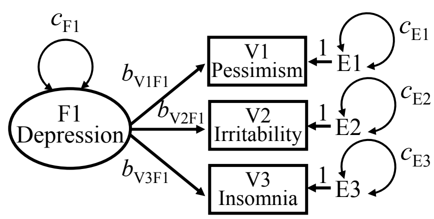
Depression:
An Unstandardized Illustration



$V1 = b_{V1F1}\,F1 + E1$

$V2 = b_{V2F1}\,F1 + E2$

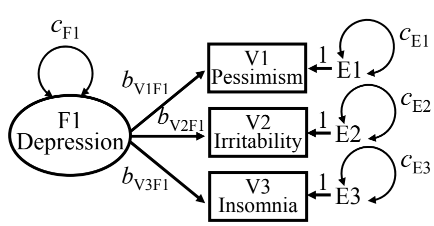
$V3 = b_{V3F1}\,F1 + E3$

Var(V1) =

Var(V2) =

Var(V3) =

Depression:
An Unstandardized Illustration



$V1 = b_{V1F1}\,F1 + E1$

$V2 = b_{V2F1}\,F1 + E2$

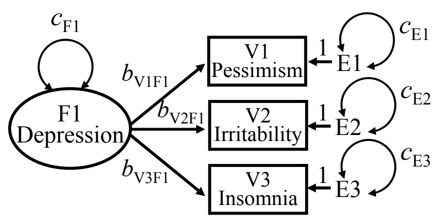
$V3 = b_{V3F1}\,F1 + E3$

Cov(V1,V2) =

Cov(V1,V3) =

Cov(V2,V3) =

Depression:
An Unstandardized Illustration



$$V1 = b_{V1F1}\,F1 + E1$$

$$V2 = b_{V2F1}\,F1 + E2$$

$$V3 = b_{V3F1}\,F1 + E3$$

Model-Implied Variances/Covariances

$$\hat{\boldsymbol{\Sigma}} = \begin{bmatrix} b_{V1F1}^2 c_{F1} + c_{E1} & & \\ b_{V1F1} b_{V2F1} c_{F1} & b_{V2F1}^2 c_{F1} + c_{E2} & \\ b_{V1F1} b_{V3F1} c_{F1} & b_{V2F1} b_{V3F1} c_{F1} & b_{V3F1}^2 c_{F1} + c_{E3} \end{bmatrix}$$

Depression:
An Unstandardized Illustration

Observed Variances/Covariances

$$\mathbf{S} = \begin{bmatrix} 100.00 & & \\ 33.00 & 121.00 & \\ 42.00 & 55.44 & 144.00 \end{bmatrix}$$

Model-Implied Variances/Covariances

$$\hat{\boldsymbol{\Sigma}} = \begin{bmatrix} b_{V1F1}^2 c_{F1} + c_{E1} & & \\ b_{V1F1} b_{V2F1} c_{F1} & b_{V2F1}^2 c_{F1} + c_{E2} & \\ b_{V1F1} b_{V3F1} c_{F1} & b_{V2F1} b_{V3F1} c_{F1} & b_{V3F1}^2 c_{F1} + c_{E3} \end{bmatrix}$$

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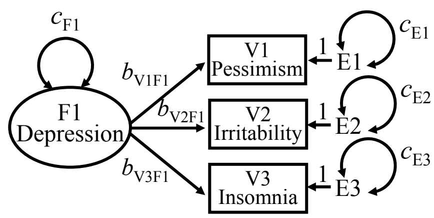
Depression:
An Unstandardized Illustration



Interpretation?

$V1 = b_{V1F1} F1 + E1$

$V2 = b_{V2F1} F1 + E2$

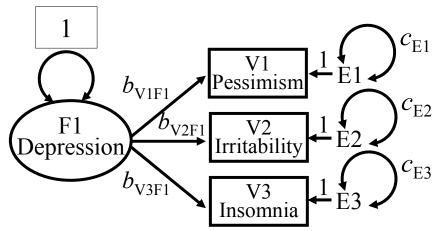
$V3 = b_{V3F1} F1 + E3$

Parameter Identification

- Necessary – but not Sufficient – Conditions
  - $t \leq u$ where
    - t = total number of parameters to be estimated (including loadings as well as modeled variances and covariances of the latent factors and of the errors)
    - $u = p(p+1)/2$ = number of unique variances and covariances of observed variables
  - Each latent variable must have an assigned unit of measurement (either by standardizing the factor or specifying a reference variable).

Depression:
An Unstandardized Illustration

Each latent variable must have an assigned unit of measurement (either by standardizing the factor or specifying a reference variable)



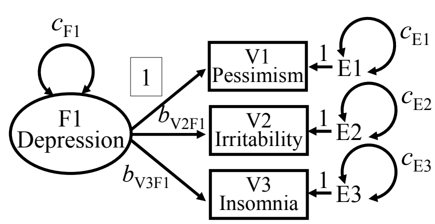
$$V1 = b_{V1F1}\, F1 + E1$$

$$V2 = b_{V2F1}\, F1 + E2$$

$$V3 = b_{V3F1}\, F1 + E3$$

Depression:
An Unstandardized Illustration

Each latent variable must have an assigned unit of measurement (either by standardizing the factor or specifying a reference variable)



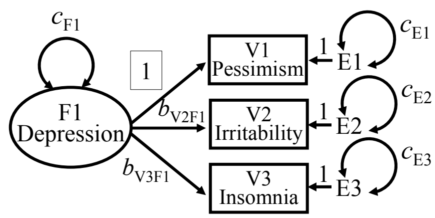
$$V1 = 1\, F1 + E1$$

$$V2 = b_{V2F1}\, F1 + E2$$

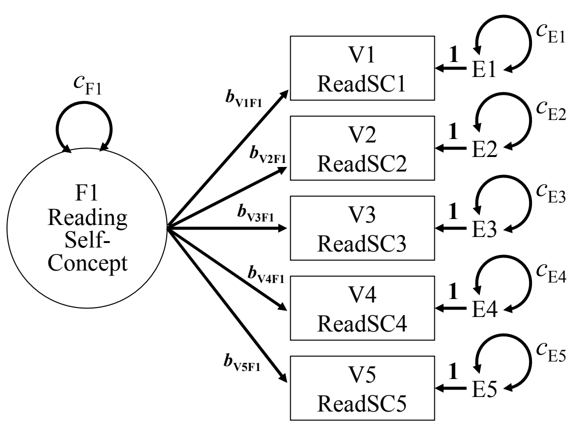
$$V3 = b_{V3F1}\, F1 + E3$$

Depression:
An Unstandardized Illustration

$u = p(p+1)/2=6$;
$t = 6$;
the model is just-identified with df=6-6=0



Reading Self-Concept:
An Unstandardized Illustration



$V1 = b_{V1F1}F1 + E1$

$V2 = b_{V2F1}F1 + E2$

$V3 = b_{V3F1}F1 + E3$

$V4 = b_{V4F1}F1 + E4$

$V5 = b_{V5F1}F1 + E5$

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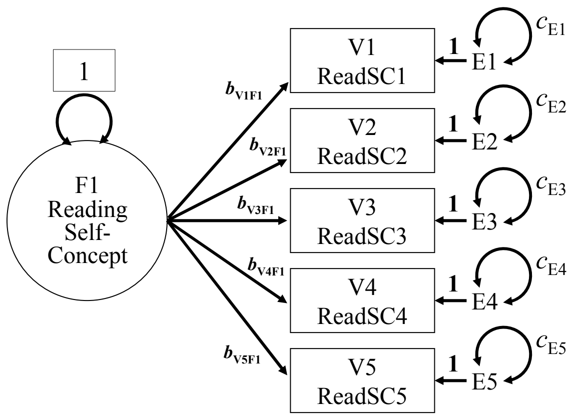
Identification:
Standardizing the Factor



$V1 = b_{V1F1}F1 + E1$

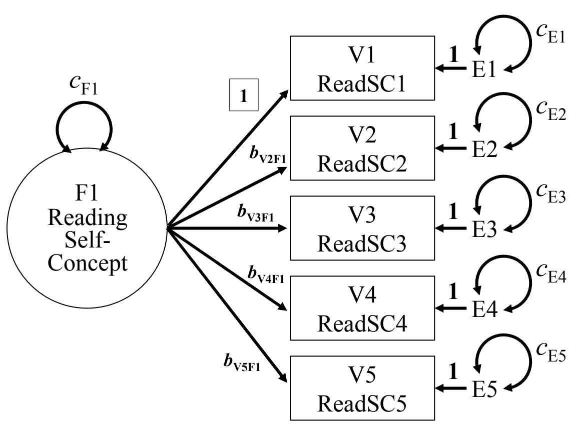
$V2 = b_{V2F1}F1 + E2$

$V3 = b_{V3F1}F1 + E3$

$V4 = b_{V4F1}F1 + E4$

$V5 = b_{V5F1}F1 + E5$

Identification:
Specifying a Reference Variable



$V1 = \boxed{1}\,F1 + E1$

$V2 = b_{V2F1}F1 + E2$

$V3 = b_{V3F1}F1 + E3$

$V4 = b_{V4F1}F1 + E4$

$V5 = b_{V5F1}F1 + E5$

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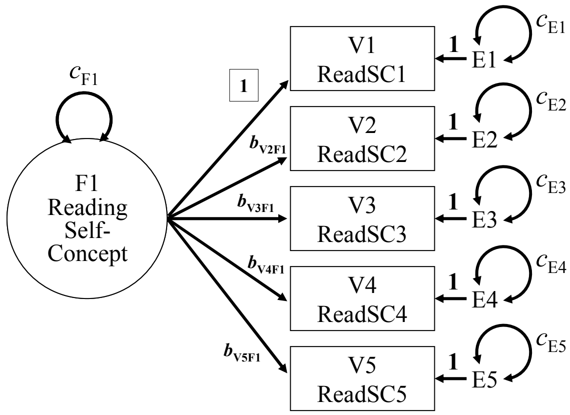
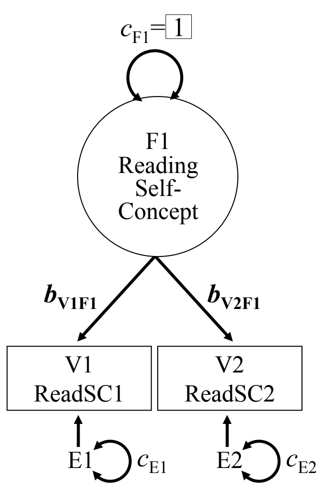
A ?-Identified One-Factor Model



***u* = *p*(*p*+1)/2=15;**
***t* = 10;**
the model is over-identified with *df*=15-10=5

A ?-Identified One-Factor Model



***u* = *p*(*p*+1)/2=3;**
***t* = 4;**
the model is under-identified

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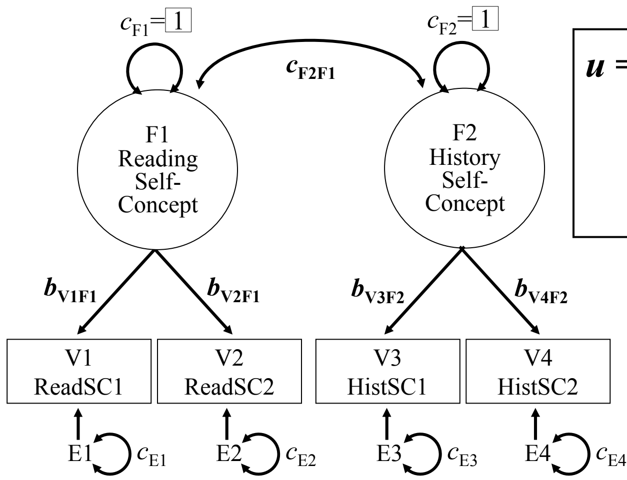
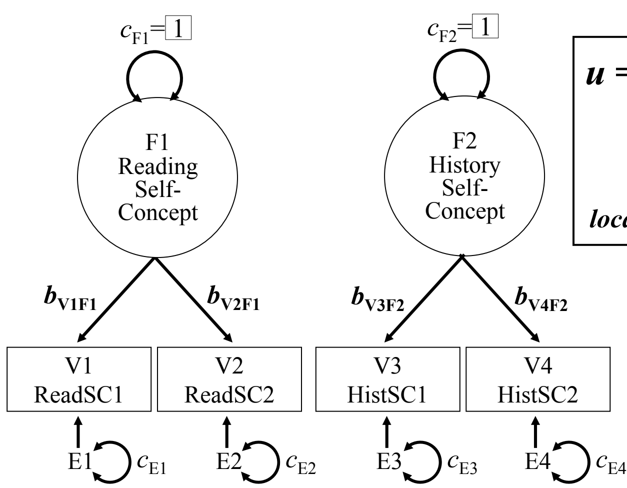
A ?-Identified Two-Factor Model



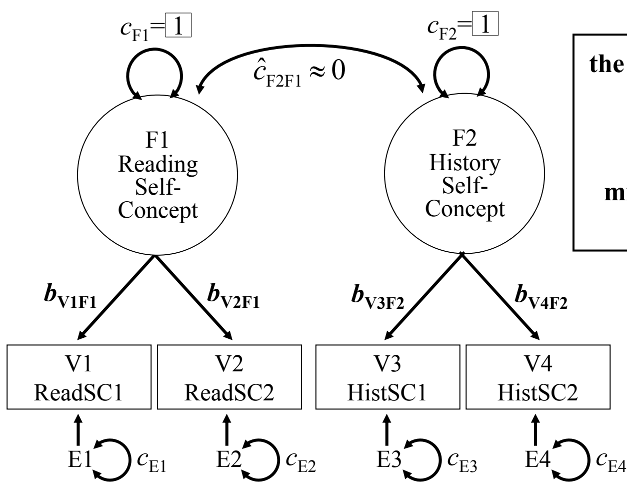
***u* = *p*(*p*+1)/2=10;**
***t* = 9;**
the model is over-identified with *df*=10-9=1

A ?-Identified Two-Factor Model



***u* = *p*(*p*+1)/2=10;**
***t* = 8;**
but the model is *locally* under-identified

A ?-Identified Two-Factor Model



the model is technically over-identified with *df*=10-9=1; but might be *empirically* under-identified

SIMPLIS:

CFA EXAMPLE

Go

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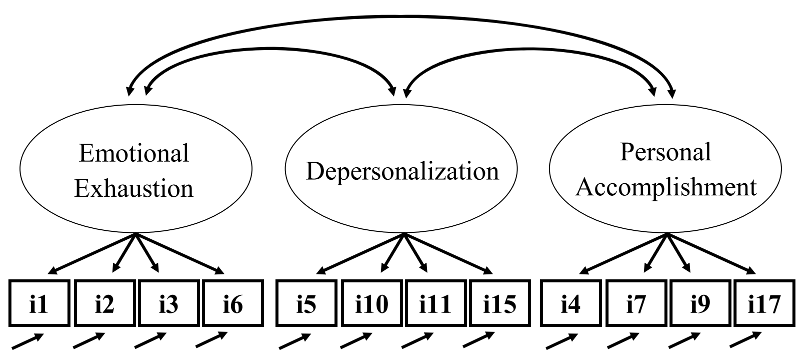
- This example utilizes SIMPLIS to assess the data-model fit of a CFA model for the Maslach Burnout Inventory (MBI; Maslach & Jackson, 1986; model and data from Byrne, 2001).
- *n* = 580 elementary teachers completed the MBI (Byrne, 1993).

The Model



Each item is a 7-point rating scale: 0='feeling has never been experienced' to 6='feeling experienced daily'

(model of the *Maslach Burnout Inventory* adapted from Byrne, 2001)

Covariance Matrix for MBI Model
(data from *n* = 580 teachers; Byrne, 2001)

	i4	i7	i9	i17	i5	i10	i11	i15	i1	i2	i3	i6
i4	0.870											
i7	0.345	0.765										
i9	0.270	0.333	1.775									
i17	0.312	0.279	0.446	0.781								
i5	-0.210	-0.301	-0.386	-0.364	2.182							
i10	-0.223	-0.200	-0.371	-0.451	0.936	2.178						
i11	-0.235	-0.234	-0.369	-0.347	0.876	1.520	2.283					
i15	-0.207	-0.236	-0.220	-0.240	0.704	0.595	0.601	1.255				
i1	-0.065	-0.095	-0.325	-0.309	0.625	0.749	0.850	0.462	2.752			
i2	-0.078	-0.085	-0.253	-0.318	0.512	0.640	0.741	0.379	1.915	2.472		
i3	-0.136	-0.226	-0.518	-0.376	0.854	0.746	0.930	0.512	1.709	1.545	2.924	
i6	-0.106	-0.253	-0.295	-0.415	1.016	0.869	0.867	0.589	1.198	1.097	1.349	2.747

SIMPLIS syntax for MBI Model

The Maslach Burnout Inventory Model
OBSERVED VARIABLES
i4 i7 i9 i17
i5 i10 i11 i15
i1 i2 i3 i6

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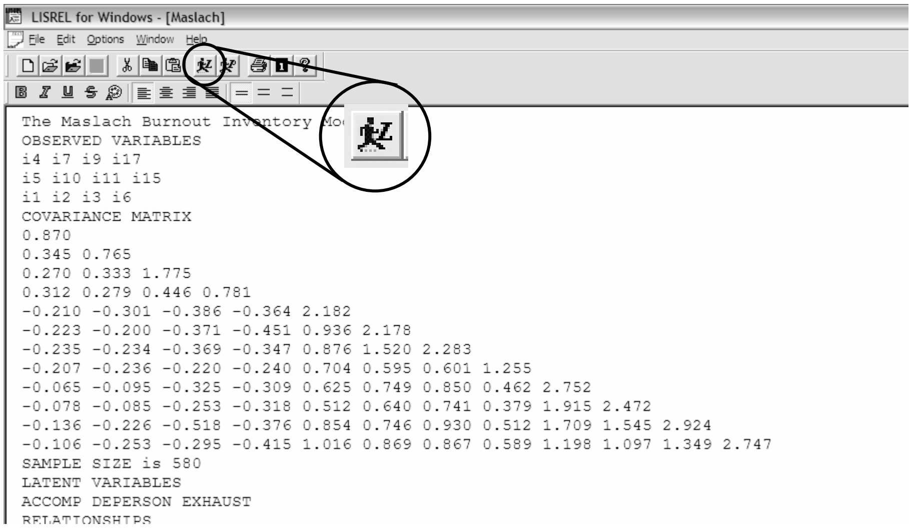
SIMPLIS syntax for MBI Model (cont.)

COVARIANCE MATRIX
0.870
0.345 0.765
0.270 0.333 1.775
0.312 0.279 0.446 0.781
-0.210 -0.301 -0.386 -0.364 2.182
-0.223 -0.200 -0.371 -0.451 0.936 2.178
-0.235 -0.234 -0.369 -0.347 0.876 1.520 2.283
-0.207 -0.236 -0.220 -0.240 0.704 0.595 0.601 1.255
-0.065 -0.095 -0.325 -0.309 0.625 0.749 0.850 0.462 2.752
-0.078 -0.085 -0.253 -0.318 0.512 0.640 0.741 0.379 1.915 2.472
-0.136 -0.226 -0.518 -0.376 0.854 0.746 0.930 0.512 1.709 1.545 2.924
-0.106 -0.253 -0.295 -0.415 1.016 0.869 0.867 0.589 1.198 1.097 1.349 2.747

SIMPLIS syntax for MBI Model (cont.)

SAMPLE SIZE is 580
LATENT VARIABLES
ACCOMP DEPERSON EXHAUST
RELATIONSHIPS
i4 i7 i9 i17 = ACCOMP
i5 i10 i11 i15 = DEPERSON
i1 i2 i3 i6 = EXHAUST
PATH DIAGRAM
END OF PROBLEM

LISREL interface for MBI Model




```
The Maslach Burnout Inventory Mo
OBSERVED VARIABLES
i4 i7 i9 i17
i5 i10 i11 i15
i1 i2 i3 i6
COVARIANCE MATRIX
0.870
0.345 0.765
0.270 0.333 1.775
0.312 0.279 0.446 0.781
-0.210 -0.301 -0.386 -0.364 2.182
-0.223 -0.200 -0.371 -0.451 0.936 2.178
-0.235 -0.234 -0.369 -0.347 0.876 1.520 2.283
-0.207 -0.236 -0.220 -0.240 0.704 0.595 0.601 1.255
-0.065 -0.095 -0.325 -0.309 0.625 0.749 0.850 0.462 2.752
-0.078 -0.085 -0.253 -0.318 0.512 0.640 0.741 0.379 1.915 2.472
-0.136 -0.226 -0.518 -0.376 0.854 0.746 0.930 0.512 1.709 1.545 2.924
-0.106 -0.253 -0.295 -0.415 1.016 0.869 0.867 0.589 1.198 1.097 1.349 2.747
SAMPLE SIZE is 580
LATENT VARIABLES
ACCOMP DEPERSON EXHAUST
RELATIONSHIPS
```

SIMPLIS output for MBI Model

```
LISREL Estimates (Maximum Likelihood)

        Measurement Equations

      i4 = 0.51*ACCOMP, Errorvar.= 0.61  , R² = 0.30
          (0.043)                (0.043)
           11.92                  13.93

      i7 = 0.50*ACCOMP, Errorvar.= 0.52  , R² = 0.33
          (0.040)                (0.038)
           12.35                  13.60

      i9 = 0.66*ACCOMP, Errorvar.= 1.34  , R² = 0.25
          (0.062)                (0.091)
           10.65                  14.73

     i17 = 0.63*ACCOMP, Errorvar.= 0.39  , R² = 0.50
          (0.041)                (0.039)
           15.45                  9.88
```

SIMPLIS output for MBI Model (cont.)

```
 i5 = 0.82*DEPERSON, Errorvar.= 1.50  , R² = 0.31
     (0.062)                  (0.097)
      13.36                    15.41

i10 = 1.20*DEPERSON, Errorvar.= 0.75  , R² = 0.66
     (0.057)                  (0.077)
      21.09                    9.70

i11 = 1.20*DEPERSON, Errorvar.= 0.84  , R² = 0.63
     (0.058)                  (0.081)
      20.61                    10.33

i15 = 0.56*DEPERSON, Errorvar.= 0.94  , R² = 0.25
     (0.048)                  (0.060)
      11.69                    15.86
```

SIMPLIS output for MBI Model (cont.)

```
 i1 = 1.42*EXHAUST, Errorvar.= 0.74  , R² = 0.73
     (0.059)                 (0.078)
      23.85                   9.48

 i2 = 1.30*EXHAUST, Errorvar.= 0.79  , R² = 0.68
     (0.057)                 (0.072)
      22.66                   10.98

 i3 = 1.23*EXHAUST, Errorvar.= 1.40  , R² = 0.52
     (0.065)                 (0.099)
      18.95                   14.09

 i6 = 0.92*EXHAUST, Errorvar.= 1.90 , R² = 0.31
     (0.068)                 (0.12)
      13.65                   15.85
```

SIMPLIS output for MBI Model (cont.)

```
Correlation Matrix of Independent Variables

               ACCOMP   DEPERSON    EXHAUST
             --------   --------   --------
   ACCOMP        1.00
 DEPERSON       -0.50       1.00
               (0.05)
               -10.83
  EXHAUST       -0.31       0.52       1.00
               (0.05)     (0.04)
                -6.18      13.50
```

Diagram output for MBI Model

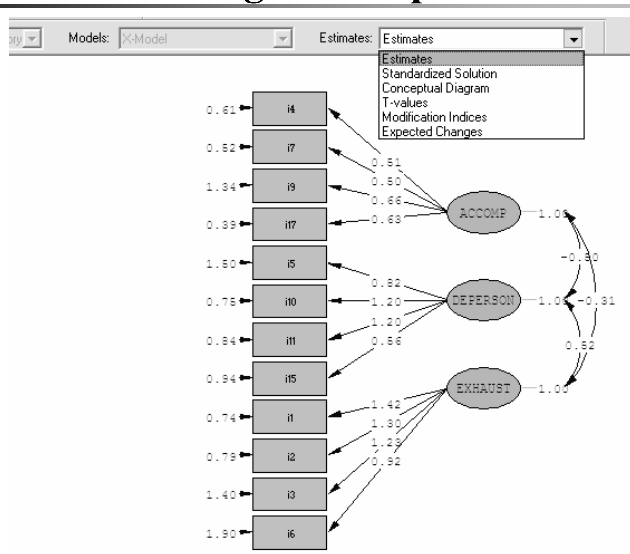
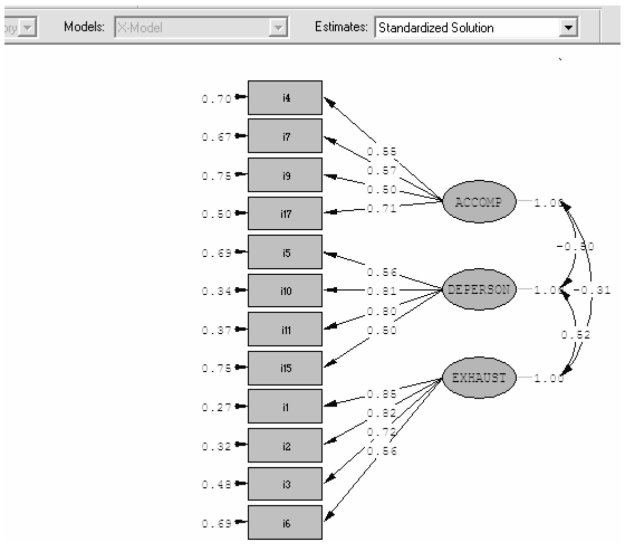


Diagram output for MBI Model (cont.)



SIMPLIS output for MBI Model (cont.)

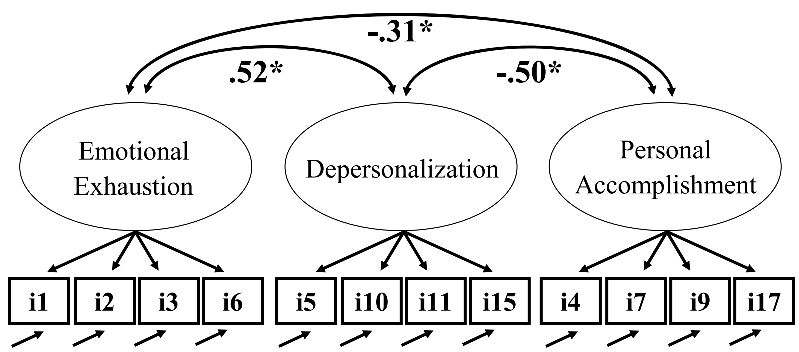
```
                    Degrees of Freedom = 51
      Minimum Fit Function Chi-Square = 248.98 (P = 0.0)
...
  Root Mean Square Error of Approximation (RMSEA) = 0.086
    90 Percent Confidence Interval for RMSEA = (0.076 ; 0.096)
       P-Value for Test of Close Fit (RMSEA < 0.05) = 0.00

                    Normed Fit Index (NFI) = 0.93
                 Non-Normed Fit Index (NNFI) = 0.93
             Parsimony Normed Fit Index (PNFI) = 0.72
                 Comparative Fit Index (CFI) = 0.95

            Root Mean Square Residual (RMR) = 0.13
                      Standardized RMR = 0.068
                  Goodness of Fit Index (GFI) = 0.93
             Adjusted Goodness of Fit Index (AGFI) = 0.89
            Parsimony Goodness of Fit Index (PGFI) = 0.61
```

χ^2 = 248.98 (*df* = 51, *p* < 0.001); SRMR = 0.068; RMSEA = 0.086, CI_{90}: (.076, 0.096); CFI = 0.95

MODEL COMPARISONS

Go

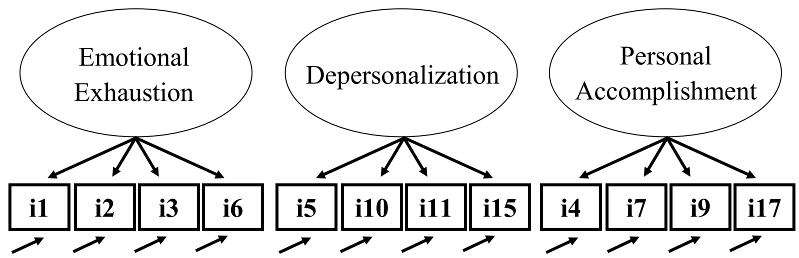
The Oblique Model



χ^2 = 248.98 (*df* = 51, *p* < 0.001);
SRMR = 0.068; CFI = 0.95;
RMSEA = 0.086, CI_{90}: (.076, .096);
AIC = 302.98

* *p* < .05

The Orthogonal Model



SIMPLIS syntax for MBI Model: Orthogonal

The Maslach Burnout Inventory Model, Orthogonal
OBSERVED VARIABLES
i4 i7 i9 i17
i5 i10 i11 i15
i1 i2 i3 i6
.
.
.
SET COVARIANCE OF ACCOMP and DEPERSON TO ZERO
SET COVARIANCE OF ACCOMP and EXHAUST TO ZERO
SET COVARIANCE OF DEPERSON and EXHAUST TO ZERO
PATH DIAGRAM
END OF PROBLEM

(SET COVARIANCES OF ACCOMP - EXHAUST TO ZERO)

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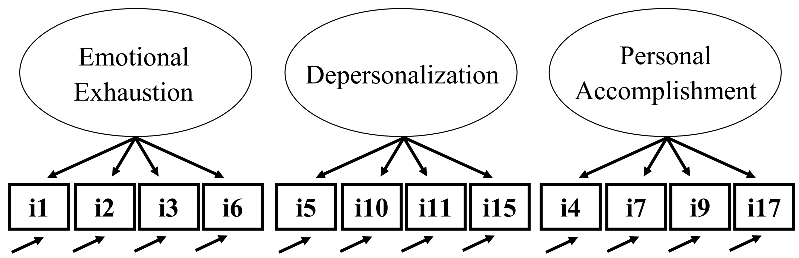
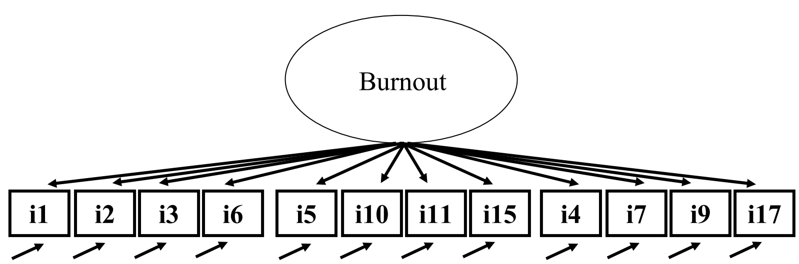
The Orthogonal Model



χ^2 = 455.62 (*df* = 54, *p* < 0.001);
SRMR = 0.19; CFI = 0.89;
RMSEA = 0.12, CI_{90}: (.11, .13);
AIC = 503.62

The One-Factor Model



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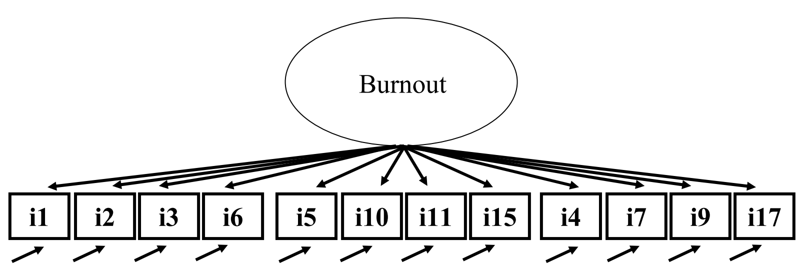
SIMPLIS syntax for MBI Model:
One-factor

The Maslach Burnout Inventory Model, One-Factor
OBSERVED VARIABLES
i4 i7 i9 i17
i5 i10 i11 i15
i1 i2 i3 i6
.
.
.
LATENT VARIABLES
BURNOUT
RELATIONSHIPS
i4 i7 i9 i17 i5 i10 i11 i15 i1 i2 i3 i6 = BURNOUT
PATH DIAGRAM
END OF PROBLEM

The One-Factor Model

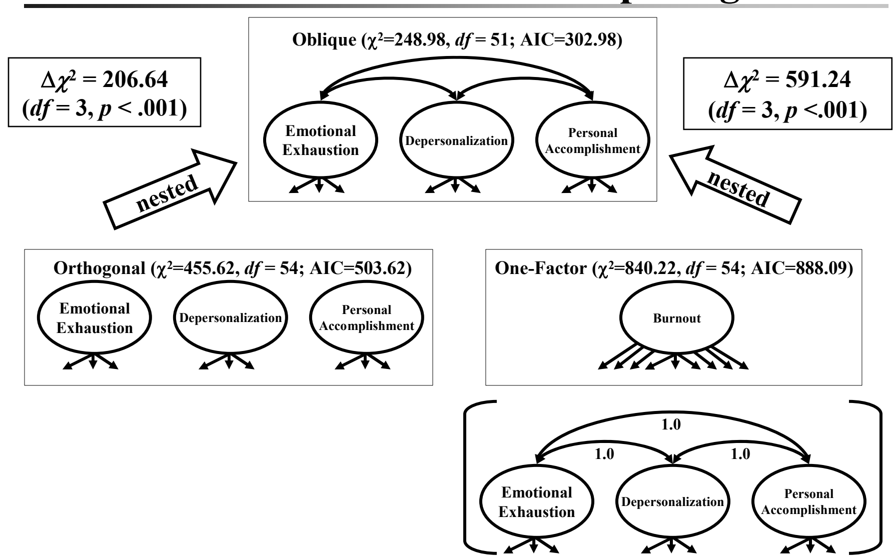


χ^2 = 840.22 (*df* = 54, *p* < 0.001);
SRMR = 0.11; CFI = 0.79;
RMSEA = 0.18, CI_{90}: (.17, .19);
AIC = 888.22

Model Comparisons

- Comparative judgments about data-model fit ("*Are the data more compatible with this or that model*?") depend on the "nested" or "non-nested" nature of the models in question.
- By their nature, such judgments are relative, not absolute: the data might not fit either model, rendering the comparison as practically meaningless.

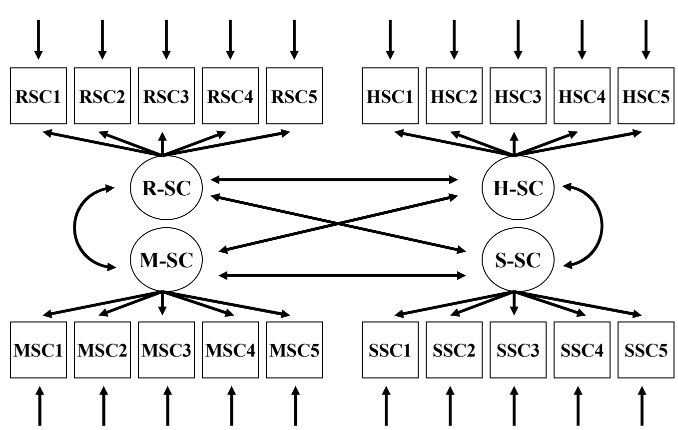
Comparing models



A Four-Factor Self-Concept Model

- Reading Self-Concept (R-SC=F1)
  - ReadSC1 (V1) – ReadSC5 (V5)
- History Self-Concept (H-SC=F2)
  - HistSC1 (V6) – HistSC5 (V10)
- Mathematics Self-Concept (M-SC=F3)
  - MathSC1 (V11) – MathSC5 (V15)
- Science Self-Concept (S-SC=F4)
  - SciSC1 (V16) – SciSC5 (V20)

A Four-Factor Self-Concept Model: The Path Diagram

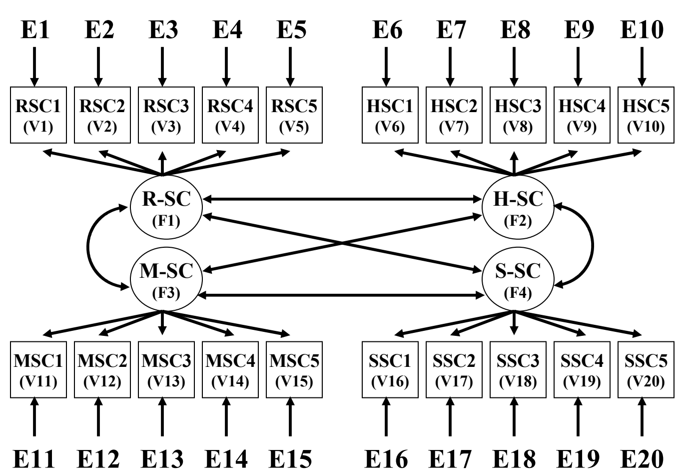


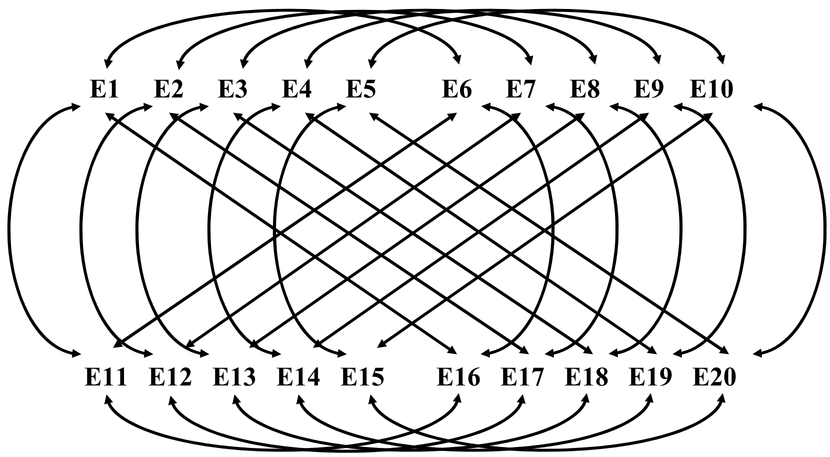
A Four-Factor Self-Concept Model:
An *a priori* addition

- All SC item sets use the same stem:

  1: "Compared to others my age, I am good at __."
  2: "I get good grades in __."
  3: "Work in __ class is easy for me."
  4: "I learn things quickly in __."
  5: "I have always done well in __."

- Because of this common item structure, error terms might covary.





SIMPLIS Syntax File

A FOUR-FACTOR SELF-CONCEPT CFA MODEL--WITH ERROR COVARIANCES

OBSERVED VARIABLES
READSC1 READSC2 READSC3 READSC4 READSC5
HISTSC1 HISTSC2 HISTSC3 HISTSC4 HISTSC5
MATHSC1 MATHSC2 MATHSC3 MATHSC4 MATHSC5
SCISC1 SCISC2 SCISC3 SCISC4 SCISC5

RAW DATA FROM FILE proficiencyraw-female.psf

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SIMPLIS Syntax File (cont.)

```
LATENT VARIABLES
READSC HISTSC MATHSC SCISC
!SIMPLIS automatically standardizes factors and assumes they covary

RELATIONSHIPS
READSC1 READSC2 READSC3 READSC4 READSC5 = READSC
HISTSC1 HISTSC2 HISTSC3 HISTSC4 HISTSC5 = HISTSC
MATHSC1 MATHSC2 MATHSC3 MATHSC4 MATHSC5 = MATHSC
SCISC1 SCISC2 SCISC3 SCISC4 SCISC5 = SCISC
```

SIMPLIS Syntax File (cont.)

```
!freeing covariances between errors of corresponding items
LET THE ERRORS BETWEEN READSC1 AND HISTSC1
CORRELATE
LET THE ERRORS BETWEEN READSC1 AND MATHSC1
CORRELATE
LET THE ERRORS BETWEEN READSC1 AND SCISC1
CORRELATE
LET THE ERRORS BETWEEN HISTSC1 AND MATHSC1
CORRELATE
LET THE ERRORS BETWEEN HISTSC1 AND SCISC1
CORRELATE
LET THE ERRORS BETWEEN MATHSC1 AND SCISC1
CORRELATE
```

SIMPLIS Syntax File (cont.)

LET THE ERRORS BETWEEN READSC2 AND HISTSC2 CORRELATE
LET THE ERRORS BETWEEN READSC2 AND MATHSC2 CORRELATE
LET THE ERRORS BETWEEN READSC2 AND SCISC2 CORRELATE
LET THE ERRORS BETWEEN HISTSC2 AND MATHSC2 CORRELATE
LET THE ERRORS BETWEEN HISTSC2 AND SCISC2 CORRELATE
LET THE ERRORS BETWEEN MATHSC2 AND SCISC2 CORRELATE
…

SIMPLIS Syntax File (cont.)

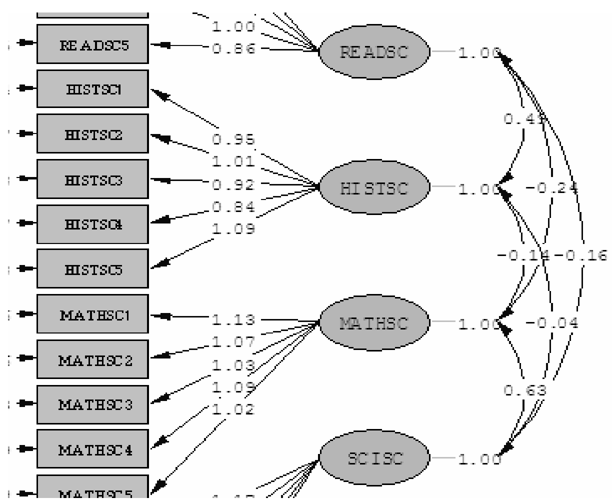
…
LET THE ERRORS BETWEEN READSC5 AND HISTSC5 CORRELATE
LET THE ERRORS BETWEEN READSC5 AND MATHSC5 CORRELATE
LET THE ERRORS BETWEEN READSC5 AND SCISC5 CORRELATE
LET THE ERRORS BETWEEN HISTSC5 AND MATHSC5 CORRELATE
LET THE ERRORS BETWEEN HISTSC5 AND SCISC5 CORRELATE
LET THE ERRORS BETWEEN MATHSC5 AND SCISC5 CORRELATE
PATH DIAGRAM
END OF PROBLEM

SIMPLIS Partial Output:
Data-model fit

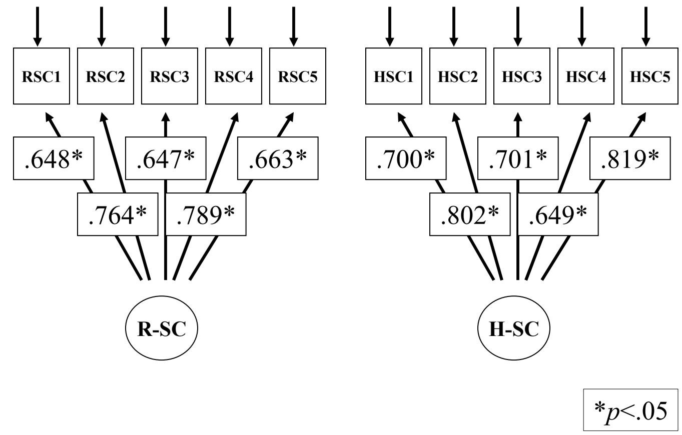
```
                    Degrees of Freedom = 134
        Minimum Fit Function Chi-Square = 167.16 (P = 0.027)
Normal Theory Weighted Least Squares Chi-Square = 165.29 (P = 0.034)

      Root Mean Square Error of Approximation (RMSEA) = 0.015
    90 Percent Confidence Interval for RMSEA = (0.0045 ; 0.023)
        P-Value for Test of Close Fit (RMSEA < 0.05) = 1.00

                       Model AIC = 317.29
                   Saturated AIC = 420.00

               Normed Fit Index (NFI) = 0.99
           Non-Normed Fit Index (NNFI) = 1.00
       Parsimony Normed Fit Index (PNFI) = 0.70
           Comparative Fit Index (CFI) = 1.00
           Incremental Fit Index (IFI) = 1.00
             Relative Fit Index (RFI) = 0.99
                   Critical N (CN) = 1046.83
       Root Mean Square Residual (RMR) = 0.032
                   Standardized RMR = 0.019
           Goodness of Fit Index (GFI) = 0.98
     Adjusted Goodness of Fit Index (AGFI) = 0.97
     Parsimony Goodness of Fit Index (PGFI) = 0.63
```

χ^2 = 167.16
(*df* = 134, *p* = 0.027);
SRMR = 0.019;
RMSEA = 0.015,
CI_{90}: (.0045, 0.023);
CFI = 1.00

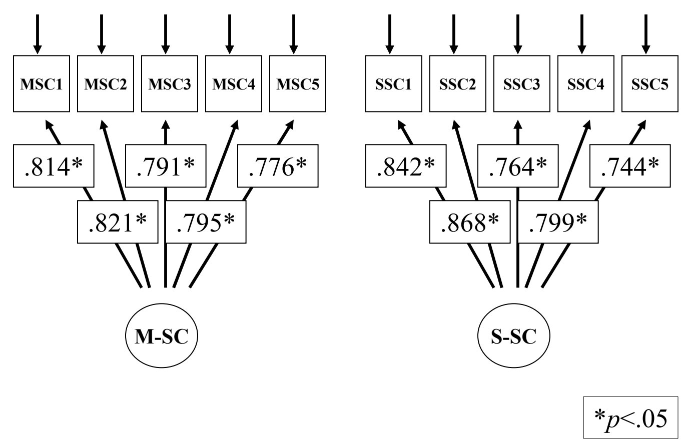
SIMPLIS Partial Output:
Unstandardized solution



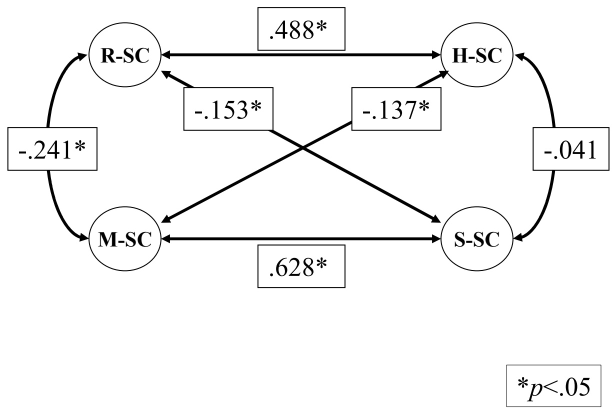
Four-Factor Self-Concept Model
Standardized Solution



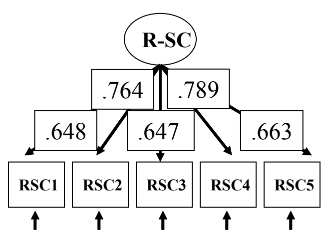
Four-Factor Self-Concept Model
Standardized Solution (cont.)



Four-Factor Self-Concept Model
Standardized Solution (cont.)

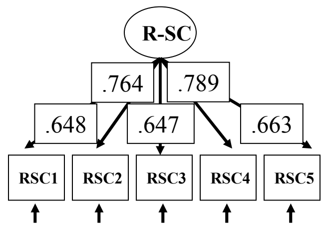


What makes a "good" factor?



- Validity
  - Does the factor "measure" what it purports to measure?
- Reliability
  - Is the factor expected to be stable and replicable?

Confirmatory Factor Analysis:
Construct Validity Considerations

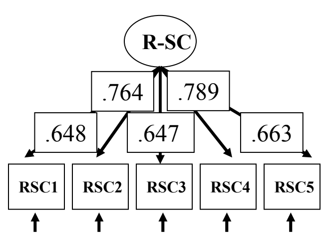


$$\text{Variance Extracted} = \sum_{i=1}^{p} \ell_i^2 \Big/ p = .497$$

(where ℓ is a standardized factor loading, and p is the number of indicators of the factor of interest.)

- Examine the sign and magnitude of the factor loadings;
- Examine sign and magnitude of inter-factor correlations;
- Conduct a multitrait-multimethod (MTMM) analysis;
- Check the Variance Extracted (some say it should exceed .50).

Confirmatory Factor Analysis:
Construct Reliability Considerations



Reliability of the Construct =
(Fornell & Larcker, 1981)

$$RC = \frac{\left(\sum_{i=1}^{p} \ell_i\right)^2}{\left[\left(\sum_{i=1}^{p} \ell_i\right)^2 + \sum_{i=1}^{p}\left(1 - \ell_i^2\right)\right]} = .832$$

- Should reliability decrease with negatively loading indicators?
- Should additional indicators detract from overall reliability?
- Should reliability be less than that of the single best indicator?

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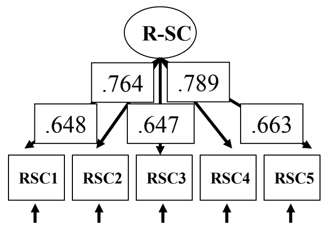
Confirmatory Factor Analysis:
Construct Reliability Considerations



Coefficient *H* ("Construct Reliability") =
(Hancock & Mueller, 2001)

$$H=\frac{1}{1+\frac{1}{\frac{\ell_1^2}{(1-\ell_1^2)}+\ldots+\frac{\ell_p^2}{(1-\ell_p^2)}}}=.841$$

- *H* is not affected by a loading's sign.
- *H* never decreases with additional indicators.
- *H* cannot be smaller than the reliability (ℓ^2) of the best indicator.

Confirmatory Factor Analysis:
Construct Reliability Considerations

- Construct Reliability (as assessed by *H*) is …

… the "stability" of a construct *as reflected in the data on the chosen indicators*

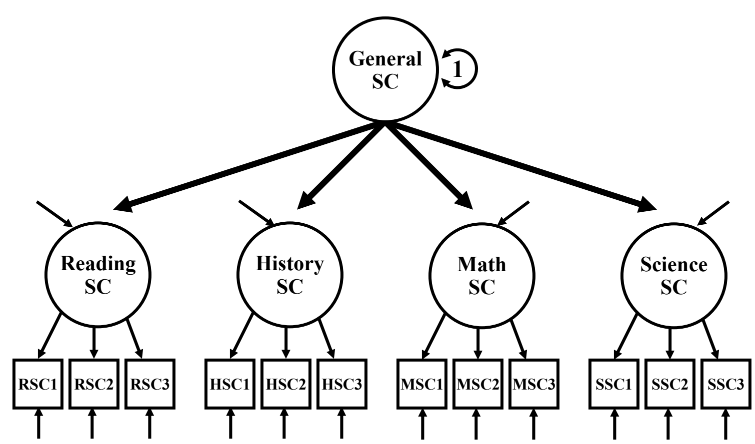
Better name might have been *Construct Replicability*

(also called "maximal reliability" in the scaling literature; it is the reliability of factor scores from the regression method of computing factor scores)

REVIEW OF OTHER CFA MODELS

Go

A 2nd Order Factor Model



plus error covariances on corresponding Self-Concept items

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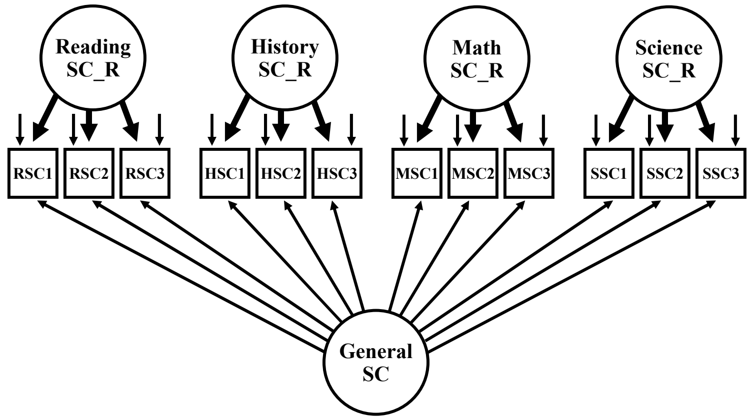
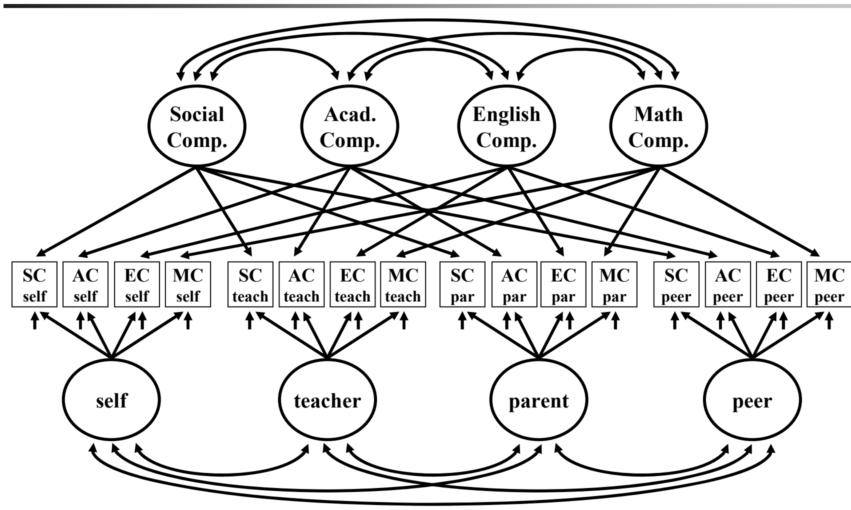
A 2nd Order Residualized Factor Model



plus error covariances on corresponding SC items

A Multitrait-Multimethod Model:
Correlated Traits / Correlated Methods



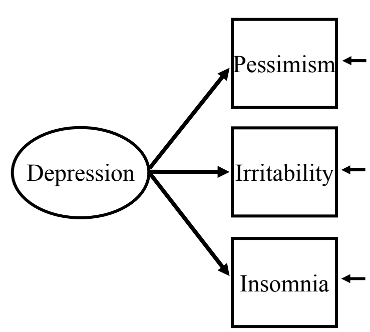
FORMATIVE MEASUREMENT MODELS

Go

Types of Measurement

I. Reflective Measurement

- *Effect* (reflective) indicators of a latent variable



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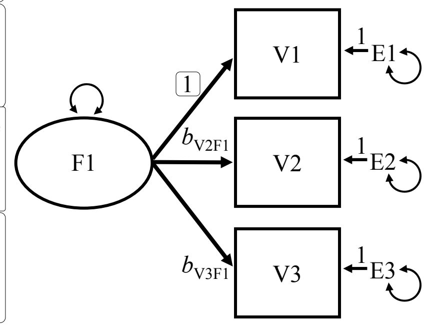
Types of Measurement

- Model equations:
  $V1 = F1 + E1$
  $V2 = b_{V2F1}F1 + E2$
  $V3 = b_{V3F1}F1 + E3$
- V1 is used as a *reference variable* to assign a metric to F1 (choice is arbitrary under equal reliability).
- Effect indicators should as a set be internally consistent (moderately high intercorrelations).
- This model is just-identified:
  $u = p(p+1)/2 = 6$ unique var/cov;
  $t = 6$ total parameters estimated.

I. Reflective Measurement

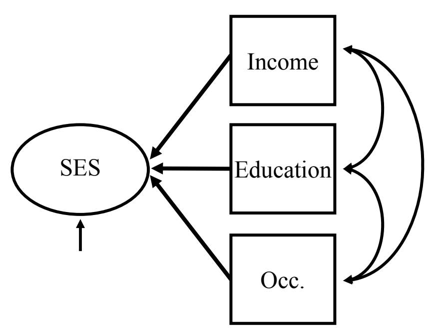
- *Effect* (reflective) indicators of a latent variable



Types of Measurement

II. Formative Measurement

- *Cause* (formative) indicators of an *emergent* variable

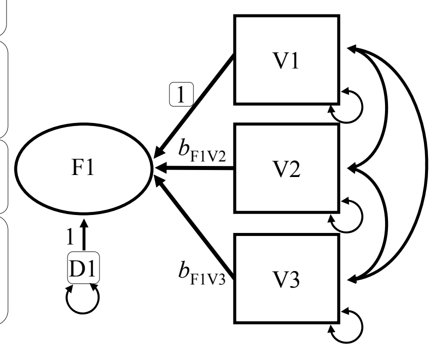


Types of Measurement

II. Formative Measurement

- *Cause* (formative) indicators of an *emergent* variable

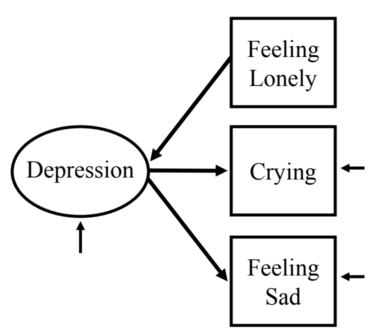
- Model equation: $F1 = V1 + b_{F1V2}V2 + b_{F1V3}V3 + D1$
- V1 provides the metric for the *latent composite* F1 (choice is arbitrary under equal reliability).
- The presence of D1 distinguishes F1 from a mere linear combination of indicators.
- As a set, cause indicators do *not* need to be internally consistent.
- This model is under-identified: $u = p(p+1)/2 = 6$ unique var/cov; $t = 9$ total parameters estimated.



Types of Measurement

III. Multiple Indicators and Multiple Causes (MIMIC)

- *Effect* & *cause* indicators

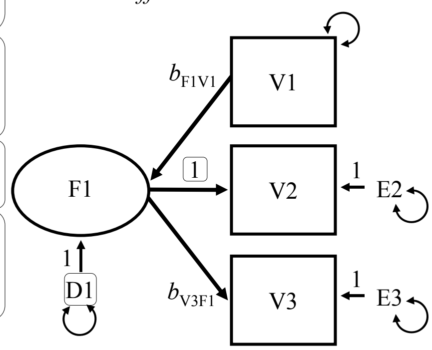


Types of Measurement

- Model equations:
  $F1 = b_{F1V1}V1 + D1$
  $V2 = F1 + E2$
  $V3 = b_{V3F1}F1 + E3$
- Either V2 or V3 should be used as a reference variable to provide the metric for F1.
- D1 reflects measurement error in the cause indicator V1.
- This model is just-identified:
  $u = p(p+1)/2 = 6$ unique var/cov;
  $t = 6$ total parameters estimated.

III. Multiple Indicators and Multiple Causes (MIMIC)

- *Effect* & *cause* indicators



Identification Conditions

- Necessary – but not Sufficient – Conditions
  - $t \leq u$ where
    t = total number of parameters to be estimated;
    $u = p(p+1)/2 =$ number of unique variances and covariances of observed variables.
  - Each latent variable must have an assigned unit of measurement.
  - In addition, the variance of a latent composite's disturbance term is not identified unless the latent composite has at least two direct effects on endogenous measured variables, or on other endogenous factors that are measured with effect indicators.

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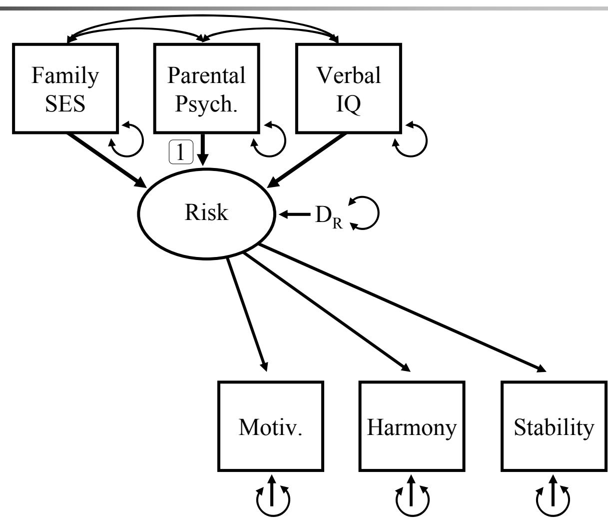
Formative Measurement Models:
Examples Using Worland et al. (1984)

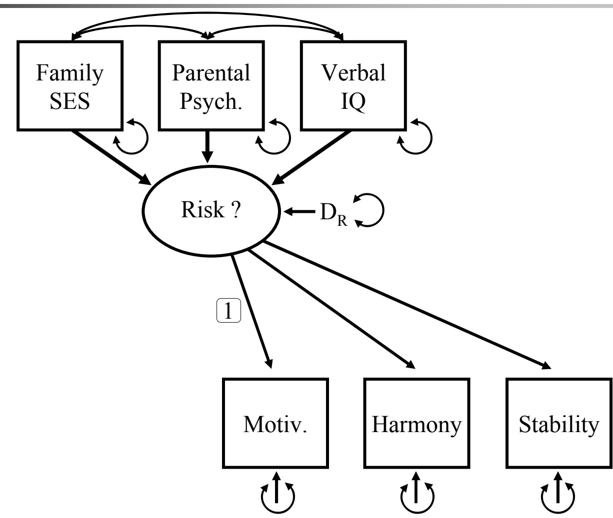
- Risk (exogenous)
  - Degree of Parental Psychopathology
  - Low Family SES
  - Verbal IQ
- Classroom Adjustment (endogenous)
  - Motivation
  - Harmony
  - Stability
- Achievement (endogenous)
  - Reading
  - Math
  - Spelling

Data (n = 158 adolescents) from Worland et al. (1984)

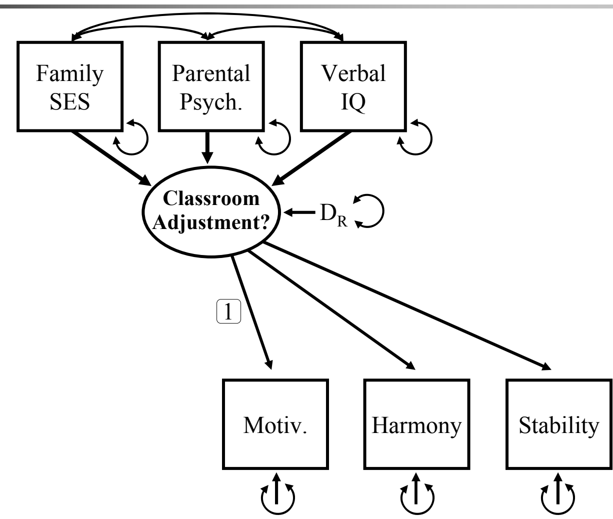
A Formative Factor Model



Not a Formative Factor Model



Not a Formative Factor Model



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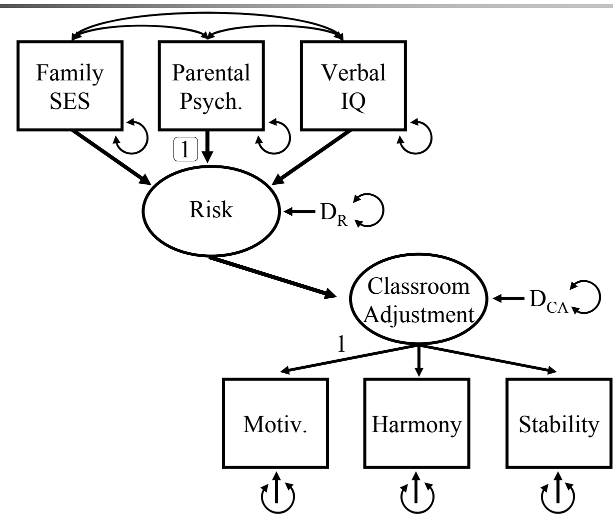
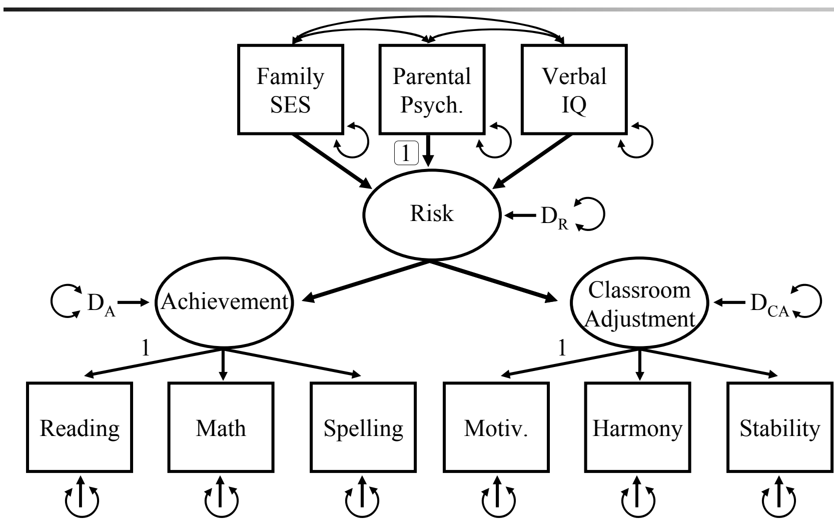
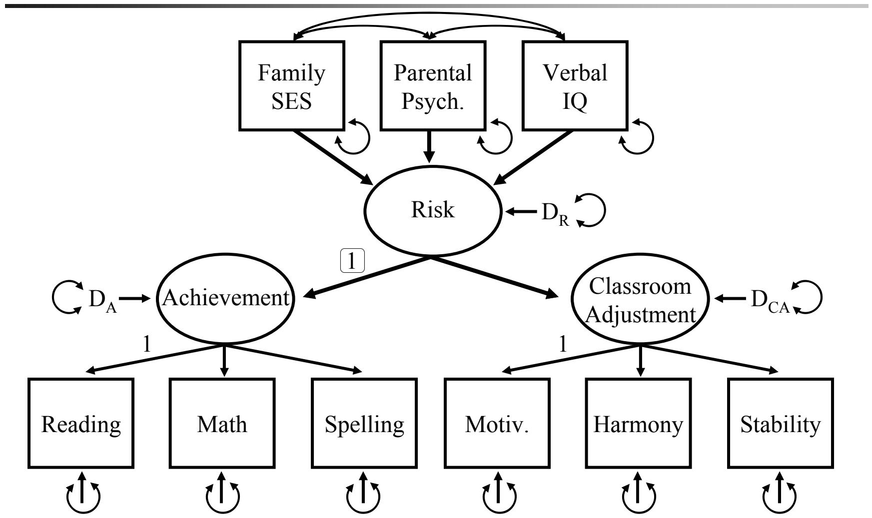
A Formative Factor Model:
Not Identified



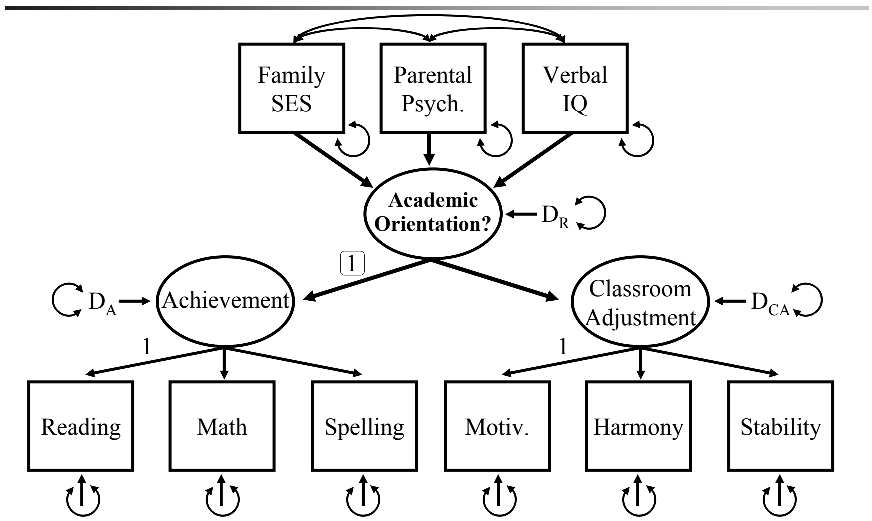
A Formative Factor Model:
Identified



Not a Formative Factor Model

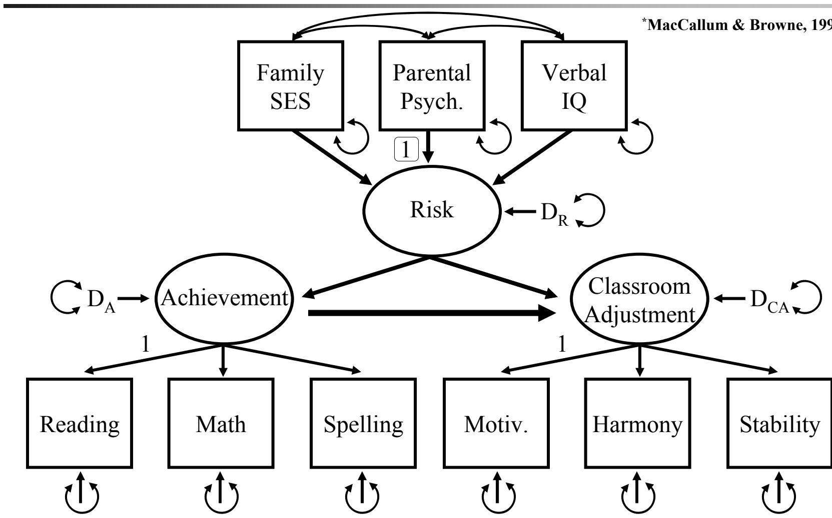


Not a Formative Factor Model



A Formative Factor Model:
Not Identified*

*MacCallum & Browne, 1993



A MIMIC Factor Model:
Identified