IRTrees for eye tracking

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Based on research in collaboration with Sun-Joo Cho and Sarah Brown-Schmidt

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**Process data**
- direct process data:
data on *activities while working on a problem*
- indirect process data:
data with relevance to inferences regarding *activities while working on a problem*

often these are parallel data: response times, brain activation 
fMRI data, EEG data, introspective questions

**Process models**
Models fitting with a process narrative 
based on direct or indirect process data
The models to be presented are
- dynamic models for direct process data they are extremely intensive longitudinal data
- they fit with a process narrative because they are tree models
The models to be presented are
- dynamic models for direct process data
  they are extremely intensive longitudinal data
- they fit with a process narrative because they
  are tree models

Which does not prove they capture real ongoing processes
we are stretching model complexity
testing the limits
to isolate the gravity effect
in the paths of for falling leafs
on a windy November day
IRT models

- extensions of discrete survival (frailty) models
- multinomial processing tree models from cognitive psychology, with random effects (Batchelder)
- have been used to model missing responses (Cees Glas)
Examples

- oatmeal
- bacon
- waffles
Wright, .., Bunge (2008)
Neural correlates of fluid reasoning
Problem solving with and without a hint
retry, learning

- Correct 1
- Incorrect 1 & 2
- Incorrect 1 correct 2
A Node 1 response is a subset of possible responses for example, “not oatmeal”

All other Node responses are *conditional responses*, responses given a condition is fulfilled for example, the Node 2 response “bacon” is a conditional response, a response conditional on “not oatmeal”

*Trees* can represent the response structure one is interested in. Only binary trees are considered (can be extended)
The conditional coding is the only one that does not induce dependency between the response options

Conditional coding

<table>
<thead>
<tr>
<th>Option 1</th>
<th>0</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td>Option 2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Option 3</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

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<td>-</td>
</tr>
<tr>
<td>Option 3</td>
<td>1</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Option 4</td>
<td>1</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>

Missingness is MAR
**The task**

Speaker instructs listener

“Click on the small ... envelope”

<table>
<thead>
<tr>
<th>pipe</th>
<th>large</th>
<th>dog</th>
</tr>
</thead>
<tbody>
<tr>
<td>balloon</td>
<td>X</td>
<td>car</td>
</tr>
<tr>
<td>small envelope</td>
<td>house</td>
<td>small elephant</td>
</tr>
</tbody>
</table>

N= 152 (listeners)
96 items in 3 conditions
112 time points
(intervals of 10 millisec between 180 and 1300ms following the onset)
pipe  large  dog
envelope
balloon  X  car
house

Node 1 = 0
Node 2 = NA
The task

Speaker instructs listener
“Click on the small ... envelope”

small envelope

small elephant

Node 1 = 1
Node 2 = 0 or 1
The task

Speaker instructs listener

“Click on the small ... envelope”

Node 1 = 1
Node 2 = 0
The task

Speaker instructs listener
“Click on the small ... envelope”

small envelope

Node 1 = 1
Node 2 = 1
Conditions in experiment

• One contrast
• Two contrasts shared
• Two contrasts privileged
One contrast

N C L
D X T
T E F

“Click on the small ... T”
Two contrasts shared

“Click on the small ... T”
Two contrasts privileged

“Click on the small ... T” listener is told that speaker does not see the large C
Coding of eye fixation on

<table>
<thead>
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<th>Node 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>other</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>competitor</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>target</td>
<td>1</td>
<td>1</td>
</tr>
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</table>

conditional response
Main interest are the fixed effects of condition coding of conditions:
- one contrast (-1) vs two contrasts shared (0.5) & two contrasts privileged (0.5)
- one contrast (0) and two con (-0.5) vs two con privileged (0.5)

Additional fixed effects
- trend
- autoregressive effects
to avoid bias in main interest estimates and standard errors

Random effects for persons and items
Three important aspects

1. Node specificity of effects
   Everything can be different between the nodes
   including multidimensionality across nodes

2. Nodes combined with random effects
   issue of selecting random effects

3. Nodes combined with time series
   two parallel series: Node 1 and Node 2
   missing observations for Node 2
1. Node specificity of effects

- Stronger positive trend for Node 2?
- Multidimensionality: a different dimension per node?
- Do condition effects depend on the node?
• Stronger positive trend for Node 2? YES
• Multidimensionality: a different dimension per node? YES, but ...
• Do condition effects depend on the node? YES
• Upward trend is steeper for within-category disambiguation than for category identification 0.031 vs 0.005

• Node 2 is a different ability compared with Node 1 
  \( r = 0.414 \)
Condition effects

Preliminary on semantics and pragmatics
Two effects based on semantics
• commonality
• contrast
One effect based on pragmatics
• knowing the conversation context and given perspective taking
The two-contrasts conditions favor Node 1

“small” is a common contrast feature of the two bottom letters
<table>
<thead>
<tr>
<th></th>
<th>Node 1</th>
<th>Node 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>two vs one</td>
<td><strong>0.074</strong> (0.014)</td>
<td>-0.370 (0.035)</td>
</tr>
<tr>
<td>two priv vs shared</td>
<td>-0.006 (0.021)</td>
<td>0.066 (0.048)</td>
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</table>
The one-contrast condition favors Node 2

“small” is a unique contrast feature of the left bottom letter
<table>
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<tr>
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<td><strong>0.074</strong> (0.014)↔</td>
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The *privileged two-contrast condition* favors Node 2

“small” is a common contrast feature of two letters but unique in the conversation context if perspective taking
<table>
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<td>two priv vs shared</td>
<td>-0.006 (0.021)</td>
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remember
2. Nodes combined with random effects

Even more random effects
Random effect selection issues: power, bias
- Minimal approach
  plus forward strategy
- Maximal approach
  plus backward strategy
- Structured search
- Sensitivity analysis focused on effects of interest
• Step 1: are nodes multidimensional?
• Step 2: if they are, then investigate model fit for random AR for persons, items, persons & items per node
• Step 3: test effects of interest with different choices for random effects
3. Nodes combined with time series

Autoregression and cross-lagged relationships for
- two time series
- missingness in the second time series
two times two series

Nodes 1 and 2
\(X_1\) : binary variable for N1 response
\[X_1 = 1 \text{ if T or C fixation (Node 1), 0 otherwise}\]
\(X_2\) : binary variable for conditional response (Node 2)
\[X_2 = 1 \text{ if T fixation, 0 if C fixation, missing if other}\]

\(X_T\) : binary variable for target fixation,
\[X_T = 1 \text{ for T fixation, 0 otherwise}\]
\(X_C\) : binary variable for competitor fixation,
\[X_C = 1 \text{ for C fixation, 0 otherwise}\]
\begin{align*}
X_T & = 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \\
X_C & = 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 1 \\
X_{N1} & = 0 \ 1 \ 0 \ 0 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 1 \\
X_{N2} & = -1 \ -1 \ -1 \ 1 \ 0 \ 1 \ 1 \ 1 \ 0 \ -0 \ 0 \\
\end{align*}

dynamic modeling which captures same information as AR1 and cross-lagged dependencies:

\begin{align*}
\text{AR1}_{T \to N1(t)} & : \text{regression of } X_{N1(t-1)} \text{ on } X_T(t-1) \\
\text{AR1}_{C \to N1(t)} & : \text{regression of } X_{N1(t-1)} \text{ on } X_C(t-1) \\
\text{AR1}_{T \to N2(t)} & : \text{regression of } X_{N2(t-1)} \text{ on } X_T(t-1) \\
\text{AR1}_{C \to N2(t)} & : \text{regression of } X_{N2(t-1)} \text{ on } X_C(t-1) \\
\end{align*}
### Fixed effects

<table>
<thead>
<tr>
<th></th>
<th>AR1 $T \rightarrow N_1 (t)$</th>
<th>AR1 $C \rightarrow N_1 (t)$</th>
<th>AR1 $T \rightarrow N_2 (t)$</th>
<th>AR1 $C \rightarrow N_2 (t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR1 $T \rightarrow N_1 (t)$</td>
<td>4.347</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR1 $C \rightarrow N_1 (t)$</td>
<td>4.024</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR1 $T \rightarrow N_2 (t)$</td>
<td>2.648</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR1 $C \rightarrow N_2 (t)$</td>
<td>-2.629</td>
<td></td>
<td></td>
<td></td>
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### Random effects

<table>
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<th>AR1 $C \rightarrow N_1 (t)$</th>
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<th>AR1 $C \rightarrow N_2 (t)$</th>
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<tbody>
<tr>
<td>AR1 $T \rightarrow N_1 (t)$</td>
<td>0.114</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR1 $C \rightarrow N_1 (t)$</td>
<td>0.748 0.340</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR1 $T \rightarrow N_2 (t)$</td>
<td>0.140 0.004 0.131</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR1 $C \rightarrow N_2 (t)$</td>
<td>-0.464 -0.515 0.788 0.247</td>
<td></td>
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Afterthoughts

• A case study of complex modeling of complex data

• A conditional response approach can be helpful to extract information/effects one wants to focus on controlling for less relevant effects in the background

• Remember the effect of two contrast privileged vs two contrast shared on Node 2 effect
  - Suppose there are individual difference in the effect (random person effects)?
  – -- they would reflect a perspective taking ability
Complex can still be beautiful