Dynamic GLMM

for Intensive Binary Temporal-Spatio Data from an Eye Tracking Technique

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- Presentation outline
  - Study motivation and purpose
  - Data description
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- Collaborators
  - Sarah Brown-Schmidt
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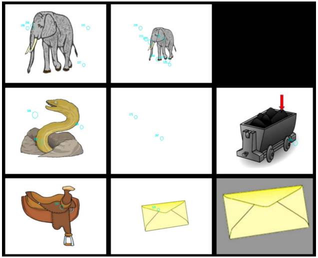
  - Paul De Boeck
    (Professor, Quantitative Psychology, Ohio State University)
- A related paper
  - Cho, S.-J., Brown-Schmidt, S., & Lee, W.-y. (2018). Autoregressive generalized linear mixed effect models with crossed random effects: An application to intensive binary time series eye-tracking data. *Psychometrika, 83,* 751-771.

Study Motivation I

- **An empirical research question**: Detecting experimental condition effects (fixed effects) from an experimental study using the visual world paradigm (Tanenhaus, Spivey-Knowlton, Eberhard,& Sedivy, 1995, *Science*).
- In the visual world paradigm, a participant hears spoken language while viewing an associated scene; the language is an external stimulus that drives eye-fixations to language-relevant interest areas in the scene.

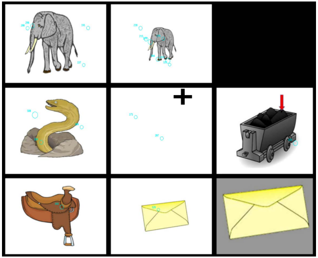


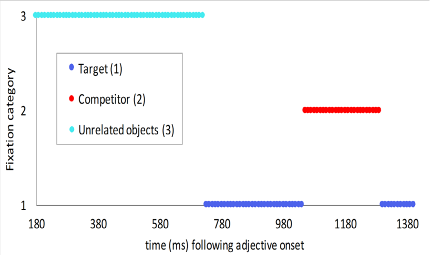
Study Motivation II

- Differences between digital and paper reading comprehension (PI: Amanda Goodwin, Associate Professor, Reading Education, Vanderbilt University)

Eye Tracking Data

- Outcome coding: Fixation data were transformed into a binary measure at each point in time of whether the listener was looking at the target or not
- Temporal information in the eye-tracking data: Intensive (equally-spaced 112 time points with 10 bins of 10 ms) time-series
- Spatial information in the eye-tracking data: Distance between the fixation point and the centroid of the target interest area





Current Practice

- Current practice to detect experimental condition effects in psycholinguistics
  - Growth curve models based on based on repeated measures
    - Ignoring various sources of dependency in experimental design factors (trial, person, item effects)
    - Hard to capture underlying functions relating time to looking behavior
  - Linear mixed models with crossed random effects (random person effect and random item effect) based on proportion measures
    - Ignoring various sources of dependency in experimental design factors (trial effect)
    - Ignoring temporal (trend and autoregressive patterns) and spatial information in the eye-tracking data
- Consequence? Biased estimates and standard errors for the fixed experimental condition effects

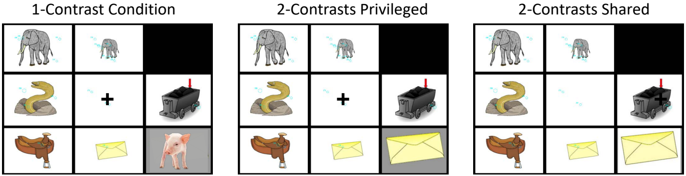
Study Purpose

- As an alternative solution: Simultaneous modeling using the smallest unit of data
  - The smallest unit of data: Binary data (whether or not a target is fixated) from trial l, person j, and item i at time t (y_{tlji})
- Study purpose: Dynamic generalized linear mixed effect model (GLMM) specification to make inference about the fixed experimental condition effects, by using random effects to take into account the complex correlation structures in eye tracking data
  - By 'dynamic' we mean that the model considers change processes (trend and autoregressive parameters) in the GLMM.

Experiment

- Experiment to test a *perspective-taking* effect with a between-subjects design
  Listener hears "Click on the small elephant"; Given 3 types of scenes:
  - *One Contrast* condition: The listener did not see a second size contrasting pair. Unambiguous (easy)
  - *Two-Contrasts Privileged* condition: The listener saw the second size contrast pair, but the big envelope was in a gray background, indicating that the speaker could not see this item. Perspective-taking (medium)
  - *Two-Contrasts Shared* condition: The listener saw a second-contrast pair such as a big envelope and a small envelope. Ambiguous (hard)

  If listeners can take into account the speakers' perspective, the Two Contrast-Privileged condition should ease target identification compared to the Two Contrast-Shared condition.



Samples and Measures

- 152 undergraduates at UIUC, all native speakers
- Each trial featured an "item", where the item was the thing participants clicked on (e.g., duck, frog, elephant). There are 96 unique items total in the data set.
- Items were repeated 3 times each, sometimes in the same condition, sometimes in another condition, sometimes on filler trials. Participants completed a total of 288 trials.
- 995,232 binary data points with 112 equally-spaced time points, 288 trials, 152 participants, and 96 items

Data Complexity

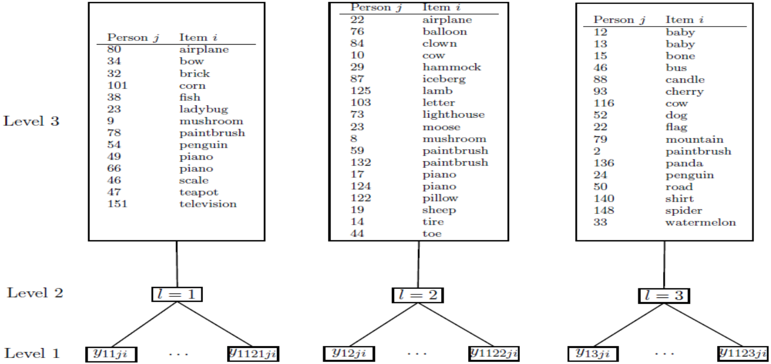


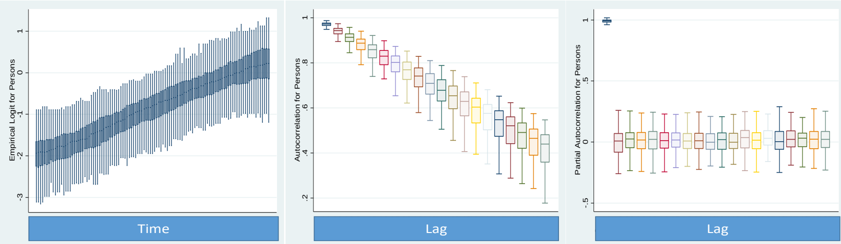
Figure 3. Data structure with selected trials.

Modeling

- Step 1: Characterizing change processes in time series data
- Step 2: Model specification
- Step 3: Model estimation
- Step 4: Model selection regarding a set of random effects
- Step 5: Model fitting and evaluation

Step 1: Characterizing Change Processes in Time Series Data

- Data description of intensive time series data based on <u>empirical logit</u> for *each* person and *each* item
  - Empirical logit measures over time: Linear trend
  - Autocorrelation: Trend + Autocorrelation (AR)
  - Partial autocorrelation: AR(1)(Millisecond-level measures of fixation position over time exhibits considerable stability.)
  - Linear growth AR(1) model: Large variability in intercepts and AR(1) effect; Small variability in trend effect



Step 2: Model Specification

- Summary of Step 1
  - Variability in intercepts and AR(1) effect across persons and across items, respectively ⇒ Random intercepts and random slopes for AR(1)
  - Little variability in linear trend effect across persons and items, respectively ⇒ Fixed trend effect
  - Similar trend patterns for all trials ⇒ Fixed trend effect
- Dynamic GLMM with crossed random effects (person random effect and item random effect)
  - Random item effect (instead of fixed item effect): Items are sampled from the item population.

Step 2: Model Specification

- Dynamic GLMM with crossed random effects is written as

$$\text{logit}[P(y_{tlji} = 1 | y_{(t-1)lji}, d_{(t-1)lji}, \mathbf{x}, \delta_{lji}, \lambda_{1j}, \rho_{1j}, \theta_j, \lambda_{2i}, \rho_{2i}\beta_i)]$$

$$= [y'_{(t-1)lji}\lambda + d'_{(t-1)lji}\rho + time'_t\zeta + \mathbf{x}'\boldsymbol{\gamma}] + \delta_{lji}$$

$$+[y'_{(t-1)lji}\lambda_{1j} + d'_{(t-1)lji}\rho_{1j} + \theta_j] + [y'_{(t-1)lji}\lambda_{2i} + d'_{(t-1)lji}\rho_{2i} + \beta_i],$$

where
- $time_t$ is a linear time covariate,
- $\mathbf{x}$ is a design matrix of fixed intercept and covariates (except the lag and trend covariates) (i.e., experimental conditions, person characteristics, item characteristics, and their interactions),
- λ is a fixed AR(1) effect,
- ρ is a fixed spatial lag effect,
- ζ is a fixed trend effect (i.e., average trend over time across persons and items),
- $\boldsymbol{\gamma}$ is a vector of fixed covariate effects (except the lag and trend covariates),
- δ_{lji} is a trial random effect (trial random intercept) for trial l, person j, and item i,
- λ_{1j} is a person AR(1) random effect (person AR(1) random slope) for person j,
- ρ_{1j} is a person spatial lag random effect (person spatial lag random slope) for person j,
- θ_j is a person random effect (person random intercept) for person j,
- λ_{2i} is an item AR(1) random effect (item AR(1) random slope) for item i,
- ρ_{2i} is an item spatial lag random effect (item spatial lag random slope) for item i, and
- β_i is an item random effect (item random intercept) for item i.

Step 2: Model Specification

- Dynamic GLMM with crossed random effects

logit[target fixation] =
[fixed effects; trend, experimental conditions,...] +
random intercept over trials +
[random slope for AR(1) over persons + random slope for spatial lag over persons + random intercept over persons] +
[random slope for AR(1) over items + random slope for spatial lag over items + random intercept over items]

Step 2: Model Specification

- An autoregressive first-order (AR1) model: y_{tlji} depends linearly on its own previous values $y_{(t-1)lji}$.
- λ_{ji} is the (model-based) conditional log odds ratio between y_{tlji} and $y_{(t-1)lji}$ (Heagerty & Zeger, 1998). The λ_{ji} is written as

$$\lambda_{ji} = log \frac{P(y_{tlji} = 1 | y_{(t-1)lji} = 1, \mathbf{x}, \delta_l, \boldsymbol{\zeta}_{1j}, \boldsymbol{\zeta}_{2i}) P(y_{tlji} = 0 | y_{(t-1)lji} = 0, \mathbf{x}, \delta_{lji}, \boldsymbol{\zeta}_{1j}, \boldsymbol{\zeta}_{2i})}{P(y_{tlji} = 1 | y_{(t-1)lji} = 0, \mathbf{x}, \delta_{lji}, \boldsymbol{\zeta}_{1j}, \boldsymbol{\zeta}_{2i}) P(y_{tlji} = 0 | y_{(t-1)lji} = 1, \mathbf{x}, \delta_{lji}, \boldsymbol{\zeta}_{1j}, \boldsymbol{\zeta}_{2i})}.$$

- The λ_{ji} is decomposed as

$$\lambda_{ji} = \lambda[\text{GrandMean}] + \lambda_{1j}[\text{VariabilityacrossPersons}] + \lambda_{2i}[\text{VariabilityacrossItems}].$$

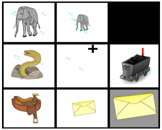
Step 2: Model Specification

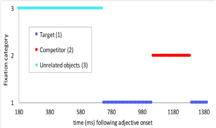
- We consider spatial information as region at target location k and region at fixation location k' for time point t, trial l, person j, and item i.

  We consider Euclidean metric for the distance between the centroids:

$$d_{tlji} = \sqrt{(n_k - n_{k'})^2 + (m_k - m_{k'})^2},$$

where n_k and $n_{k'}$ are the longitude coordinates, and m_k and $m_{k'}$ are the latitude coordinates for the target location k and the fixation location k', respectively.





Step 2: Model Specification

- Our modeling is on a probability of target fixation at a time point t. We hypothesize that the probability of having target fixation can be higher as fixation at a previous time point $t-1$ is close to the target. Thus, lag spatial covariate (e.g., $d_{(t-1)lji}$ for the first order) was considered.
- The ρ_{ji} is decomposed as

$$\rho_{ji} = \rho[\mathrm{GrandMean}] + \rho_{1j}[\mathrm{VariabilityacrossPersons}] + \rho_{2i}[\mathrm{VariabilityacrossItems}].$$

Step 3: Estimation

- The missing initial response problem for models having random intercepts
  - When the model applies to all time points, random intercepts have a direct affect on $y_{(t-1)lji}$. The random intercepts cannot be statistically independent of $y_{(t-1)lji}$ if they affect $y_{(t-1)lji}$.
- Initial time point in the current study
  - The lag response y_{0lji} was treated as a missing variable and its subsequent response variable y_{1lji} at $t = 1$ was not modelled.

Time t	y_{tlji}[Outcome]	$y_{(t-1)lji}$[AR(1) Covariate]
0[180ms]	0	.
1[190ms]	1	0
2[200ms]	1	1
3[210ms]	0	1
4[220ms]	1	0
5[230ms]	1	1
6[240ms]	1	1

Step 3: Estimation

- Justification
  - 1. There are a large number of time points (112 time points). The missing initial response problem for models having random effects and autoregressive responses can be less severe when there are a large number of time points (Hsiao, 2003).
  - 2. There is unlikely to be much change between 180 ms and 220 ms (the first 40 ms of data). The time window is offset by 200 ms due to the time needed to program an eye movement.

Time t	y_{tlji} [Outcome]	$y_{(t-1)lji}$ [AR(1) Covariate]
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5[230ms]	1	1
6[240ms]	1	1

Step 3: Estimation

- The marginal likelihood for the model described is as follows:

$$\prod_{j=1}^{J}\prod_{i=1}^{I}\int_{\boldsymbol{\zeta}_{1j}}\int_{\boldsymbol{\zeta}_{2i}}\left[\prod_{t=2}^{T-1}\prod_{l=1}^{L}\left\{\int_{\delta_{lji}} P(y_{tlji}|y_{(t-1)lji}, d_{(t-1)lji}, \mathbf{x}, \delta_{lji}, \boldsymbol{\zeta}_{1j}, \boldsymbol{\zeta}_{2i})g_1(\delta_{lji})d\delta_{lji}\right\}\right]d\boldsymbol{\zeta}_{1j}d\boldsymbol{\zeta}_{2i}\cdot$$

$$\prod_{j=1}^{J}\int_{\boldsymbol{\zeta}_{1j}} g_2(\boldsymbol{\zeta}_{1j})d\boldsymbol{\zeta}_{1j}\cdot\prod_{i=1}^{I}\int_{\boldsymbol{\zeta}_{2i}} g_3(\boldsymbol{\zeta}_{2i})d\boldsymbol{\zeta}_{2i}, \quad (1)$$

where $\boldsymbol{\zeta}_{1j} = [\theta_j, \rho_{1j}, \lambda_{1j}]'$ are person random effects, $\boldsymbol{\zeta}_{2i} = [\beta_i, \rho_{2i}, \lambda_{2i}]'$ are item random effects, $g_1(.)$ is normal density, and $g_2(.)$ and $g_3(.)$ are multivariate normal density.

- Estimation methods
  - Laplace approximation implemented in the `lme4` package version 0.999375-39 (Bates, Maechler, & Bolker, 2011) in R-2.10.1 (R Development Core Team, 2009)
  - Hierarchical Bayesian analysis implemented in Stan (Carpenter et al., 2017)

Step 4: Model Selection

- Step 4: Model selection regarding a set of random effects in using Laplace approximation

A baseline model for model comparisons:

$$\eta_{tlji} = \gamma_1 + d'_{(t-1)lji}\rho + y'_{(t-1)lji}\lambda + time'_t\gamma_2 + \delta_{lji} + \theta_j + \beta_i \quad (2)$$

	Trial	Person			Item	
Model	Trial Int.	Person Int.	Pair Int.	Slope	Item Int.	Slope
Baseline	√	√			√	
Model A	√	√	√		√	
Model B-Person	√	√		√	√	
Model B-Item	√	√			√	√
Model B	√	√		√	√	√

- Likelihood ratio test (LRT)

H_0: the variance of random intercepts $\neq 0$, variance of a random slope=covariance between a random intercept and a random slope $= 0$ vs.

H_1: variance of a random slope=covariance between a random intercept and a random slope $\neq 0$

For testing H_0, we use the mixtures of χ^2 distribution, $0.5\chi^2_1 + 0.5\chi^2_2$.

- Marginal Akaike information criterion (AIC; e.g., Greven & Kneib, 2010)
- Bayesian information criterion (BIC; Schwarz, 1978; Cho & De Boeck, 2018)

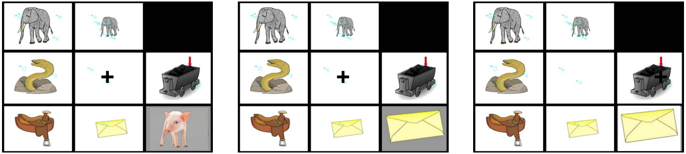
Step 5: Model Fitting and Model Evaluation

- Adding experimental condition covariates to Model B
  Helmert coding

Condition	*Contrast* Covariate	*Privileged* Covariate
One-Contrast	-1	0
Two Contrasts-Privileged	0.5	0.5
Two Contrasts-Shared	0.5	-0.5

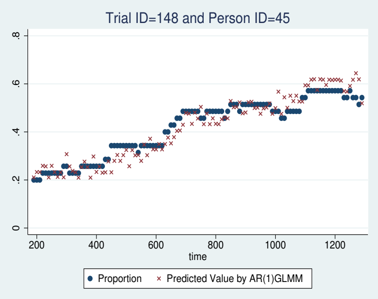
- The critical comparison was whether the Two Contrasts-Shared condition differed from the Two Contrasts-Privileged condition. If listeners can take into account the speakers' perspective, the Two Contrast-Privileged condition should ease target identification compared to the Two Contrast-Shared condition.

1-Contrast Condition 2-Contrasts Privileged 2-Contrasts Shared



Step 5: Model Fitting and Model Evaluation

- Model evaluation
  - Model-data fit: Small standardized residuals for trial l, person j, and item i at time t
  - Measure of the ordinal predictive power of the model: High somers' rank correlation between a variable $P(y_{tlji}|y_{(t-1)lji}, d_{(t-1)lji}, \mathbf{x}, \delta_{lji}, \zeta_1, \zeta_2)$ and a binary y_{tlji} (0.991)



Results

- Results for experimental condition effects
  - The significant *Contrast* effect (EST=-0.049, SE=0.015, $z = -3.21$, p-value=0.000134): Listeners were less likely to fixate the target when they were in one of the Two Contrast conditions than in the One Contrast condition.
  - The insignificant *Privileged* effect (EST=0.037, SE=0.024, z=1.55, p-value=0.12163): Does not support that listeners were more likely to fixate the target when they were in the Two Contrast-Privileged condition than when they were in the Two Contrast-Shared condition (perspective-taking effect).
- Controlling factors
  - Trend: Significant linear trend (weak)
  - Autoregression: Significant AR(1) and variability in AR(1) effects across persons and items (strong)
  - Distance: Significant distance (weak)
  - Trial, person, and item clustering: Non-ignorable dependency

Simulation Study

- Parameter recovery: Relatively satisfactory
- Consequences of ignoring autocorrelation and trend in time series data: Biased estimates and underestimated standard errors of experimental condition effects

Summary

- The proposed methods provide a new perspective for the analysis of intensive binary data from eye tracking. To our knowledge, the GLMM specification we propose is the first attempt to model various sources of variability and dependency from eye-tracking data.
- We anticipate the proposed stepwise approach to model building will enhance data analysis practice.

End

- Questions? sj.cho@vanderbilt.edu
- A related paper
  - Cho, S.-J., Brown-Schmidt, S., & Lee, W.-y. (2018). Autoregressive generalized linear mixed effect models with crossed random effects: An application to intensive binary time series eye-tracking data. *Psychometrika, 83,* 751-771. [Supplemental materials can be found at Open Science Framework website: https://osf.io/fz9j6/.]