The Effects of a Formative Assessment Intervention on Student Understanding of Basic Mathematical Principles

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Formative Assessment Continuum

On-the-fly formative assessment

Informal, unplanned. Arises when a teachable moment unexpectedly occurs and the teacher pursues it

Planned-for-interaction formative assessment

Deliberate (e.g., questions in the lesson plan intended to reveal student understanding, with use of responses to adjust instruction)

Embedded-in-the-curriculum formative assessment

Formal "tests" placed in the curricular sequence at "joints" where an important sub-goal occurs

Used to further student understanding and to adjust instruction

(Shavelson, 2008)

Formative Assessment Practices

Formative assessment often described as a process, and one which is domain-independent, and built around general classroom practices:

- Sharing learning expectations
- Questioning
- Feedback
- Peer assessment
- Self assessment

Embedded-in-the-curriculum formative assessment

Formal “tests” placed in the curricular sequence at “joints” where an important sub-goal occurs

Used to further student understanding and to adjust instruction

(Shavelson, 2008)

Assessment instrument plays a key role

Assessment is specific to the domain and focused on core understandings or big ideas in the subject area

The process around using such curriculum-embedded assessments is still a key consideration

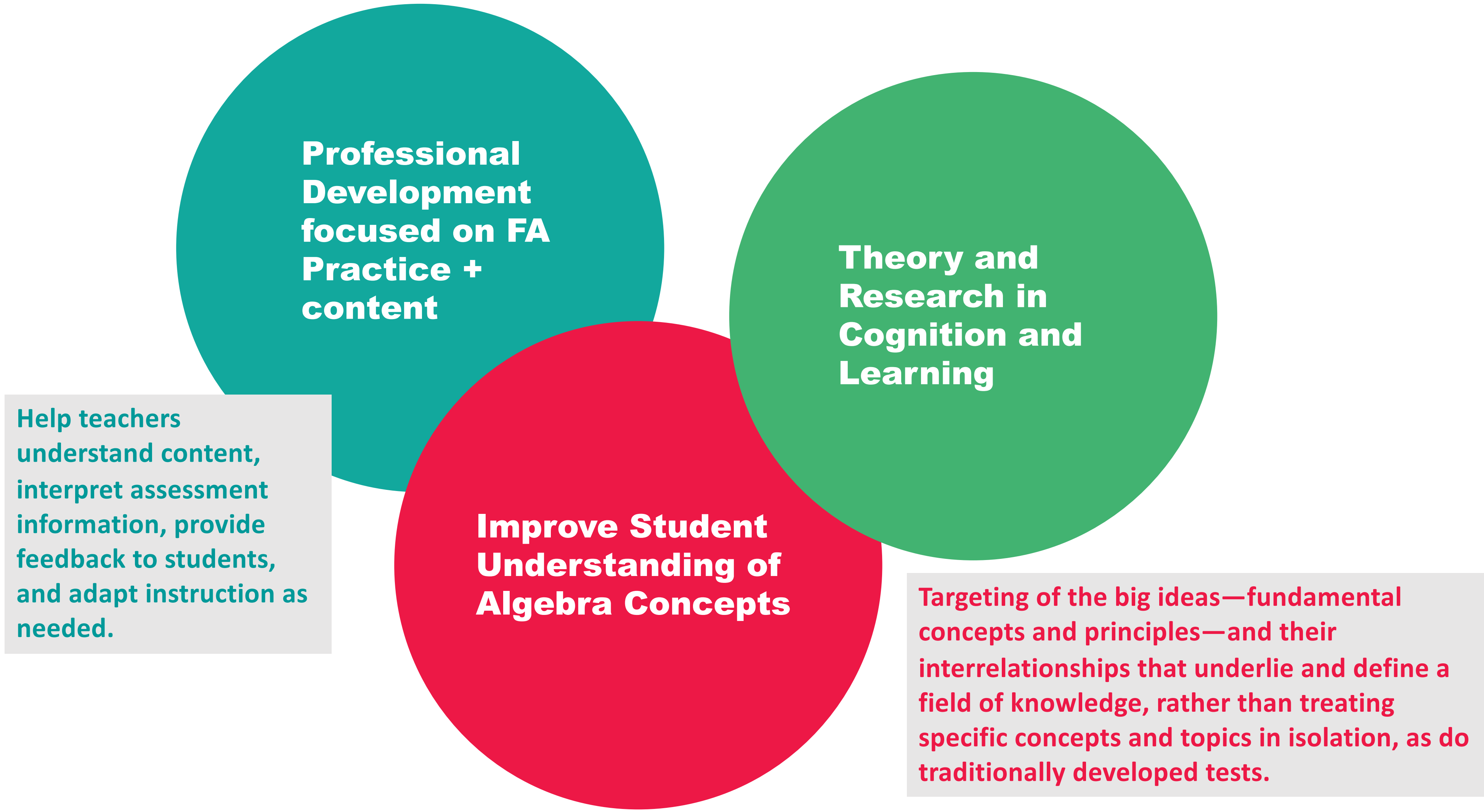
If formative assessment is seen as a domain-independent process, associated PD may focus on more generic pedagogical knowledge, or good teaching practice.

Shift in focus to developing pedagogical content knowledge

If teachers don’t have domain expertise, how can they…

- ✓ **Where are my students in relation to my goals?**
- ✓ **Have students progressed as I would expect?**
- ✓ **Do they hold any misconceptions?**
- ✓ **How can I help students to bridge the gap between where they currently are and where I want them to be?**
- ✓ **What are next steps for teaching and learning?**
- ✓ **What kinds of instructional activities will best respond to individual students' learning needs?**

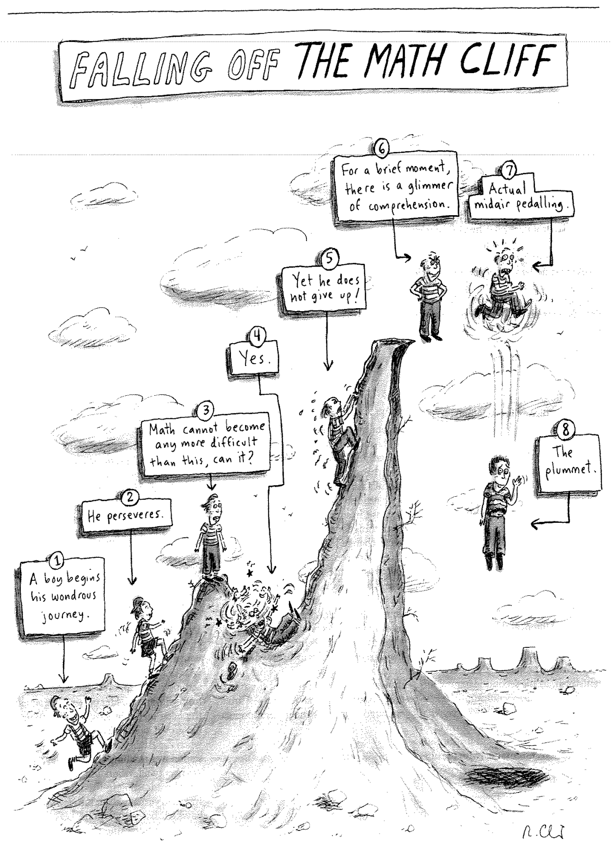
(Herman, 2013, 2016; Heritage, 2010)


Professional Development focused on FA Practice + content
Theory and Research in Cognition and Learning
Improve Student Understanding of Algebra Concepts
Help teachers understand content, interpret assessment information, provide feedback to students, and adapt instruction as needed.
Targeting of the big ideas—fundamental concepts and principles—and their interrelationships that underlie and define a field of knowledge, rather than treating specific concepts and topics in isolation, as do traditionally developed tests.

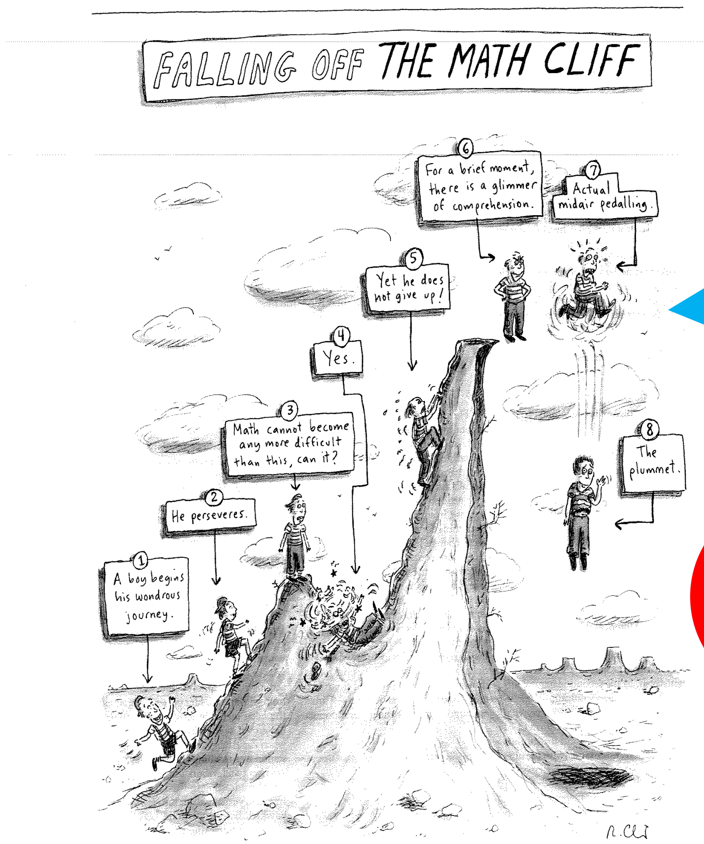
ALGEBRA CHALLENGE

- Failure to master algebra is a major roadblock to high school graduation in most states.
- Many students never master algebra and aren't able to move on in high school or college.
- Many students don't have the mastery of elementary math necessary for success in algebra.
- They don't understand the concepts and principles underlying the procedures they've been taught.
- Procedures learned without understanding don't transfer and don't provide a foundation for future learning.
- It's difficult for students who are significantly behind to catch up.

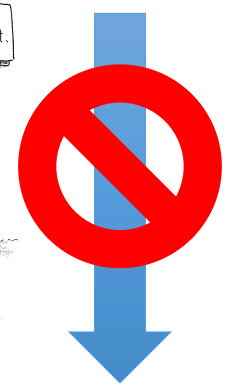
Student Need



Student Need



Intervention

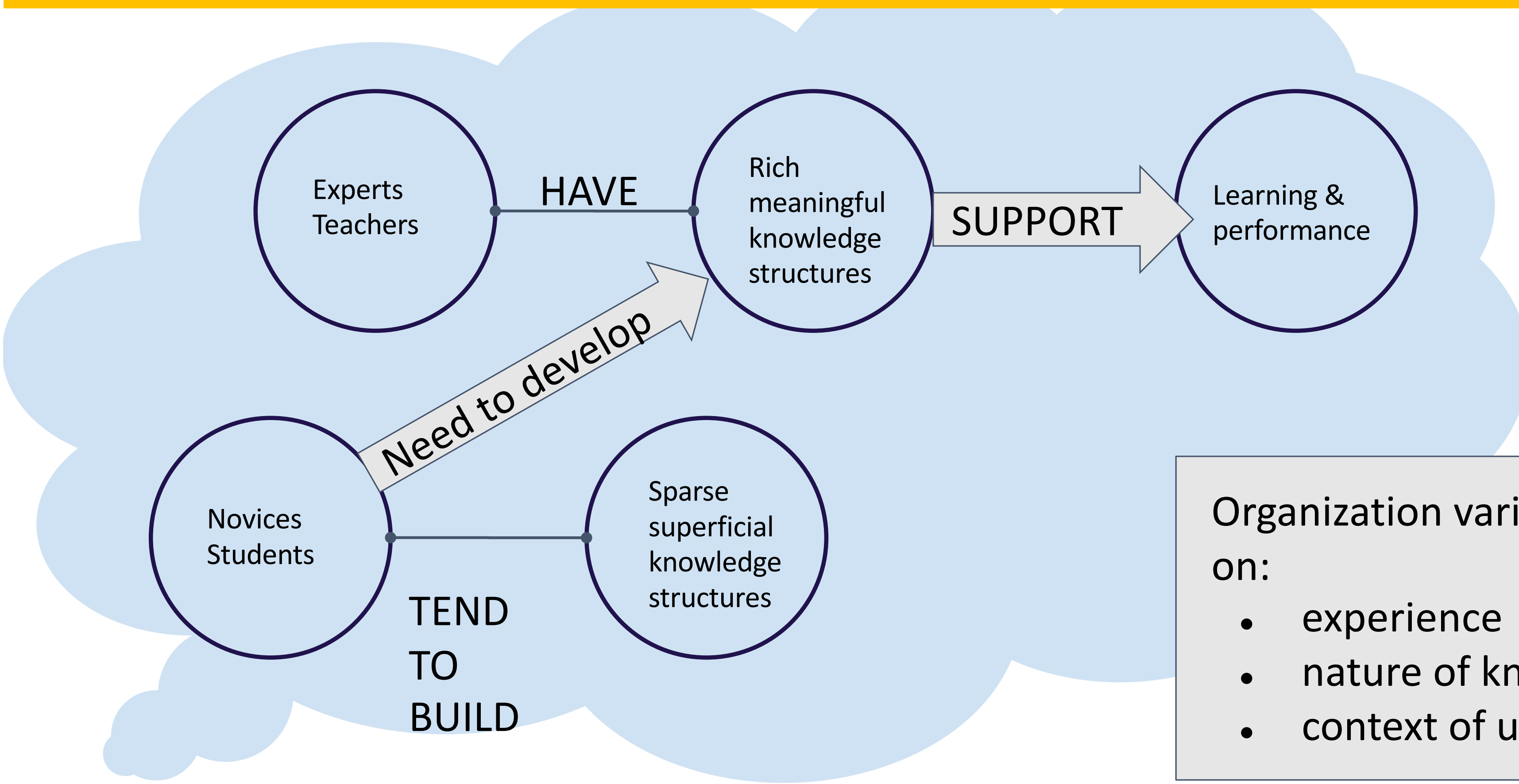


Theory and Research in Cognition and Learning

What characterizes expertise?

- Connections among elements of knowledge
- Understanding of the meaning of important concepts and principles
- Ability to apply knowledge flexibly and effectively in a wide variety of situations (transfer)

Organization of Knowledge



Organization varies based on:

- experience
- nature of knowledge
- context of use

Big Ideas

- Unfortunately, curricula and instruction do not always reflect what we know about subject area knowledge and how it develops.
- Students are often taught in a way that leads them to believe that learning means acquiring a huge number of meaningless facts and skills.
- Many students do not have a strong grasp of the most important content/big ideas

Research on Cognition and Learning

Absolute Value AddSub Both Sides Adding Fractions Addition Additive
Identity Additive Inverse Associative Property of Addition Ass
Property of Multiplication Commutative Property of Additio
Multiplication Completing the Square Distributive Prope nt
Expression Factoring Fractions Functions GCF Intege
Multiplicative Identity Multiplicative Inverse Multiplyi
Both Sides Prime Factors Rational Numbers Ratios
Second Order Equations Simplifying Square of Diff
Subtraction Symmetric Property Systems of Equa
Variables Whole Numbers Absolute Value Add Bo
Addition Additive Identity Additive Inverse Associ
Commutative Prop Add Commutative Prop Mult C
Distributive Property Equality Evaluation Expone
Fractions Functions GCF Graphing Intersection L
Multiplication Multiplicative Identity Multiplicative
Operations Power of Both Sides Prime Factors Rat
Reflexive Property Second Order Equations Simplify
Substitution Symmetric Property Systems of Equatio
Variables

- What is the relative importance of these concepts?
- Which concepts are organizing principles or big ideas?
- Which concepts are fundamental to the domain?
- Which concepts form the basis for understanding the domain?

Big Ideas

- Unfortunately, curricula and instruction do not always reflect what we know about subject area knowledge and how it develops.
- Students are often taught in a way that leads them to believe that learning means acquiring a huge number of meaningless facts and skills.
- Many students do not have a strong grasp of the most important content/big ideas
- Ontologies provide a representation of a domain.

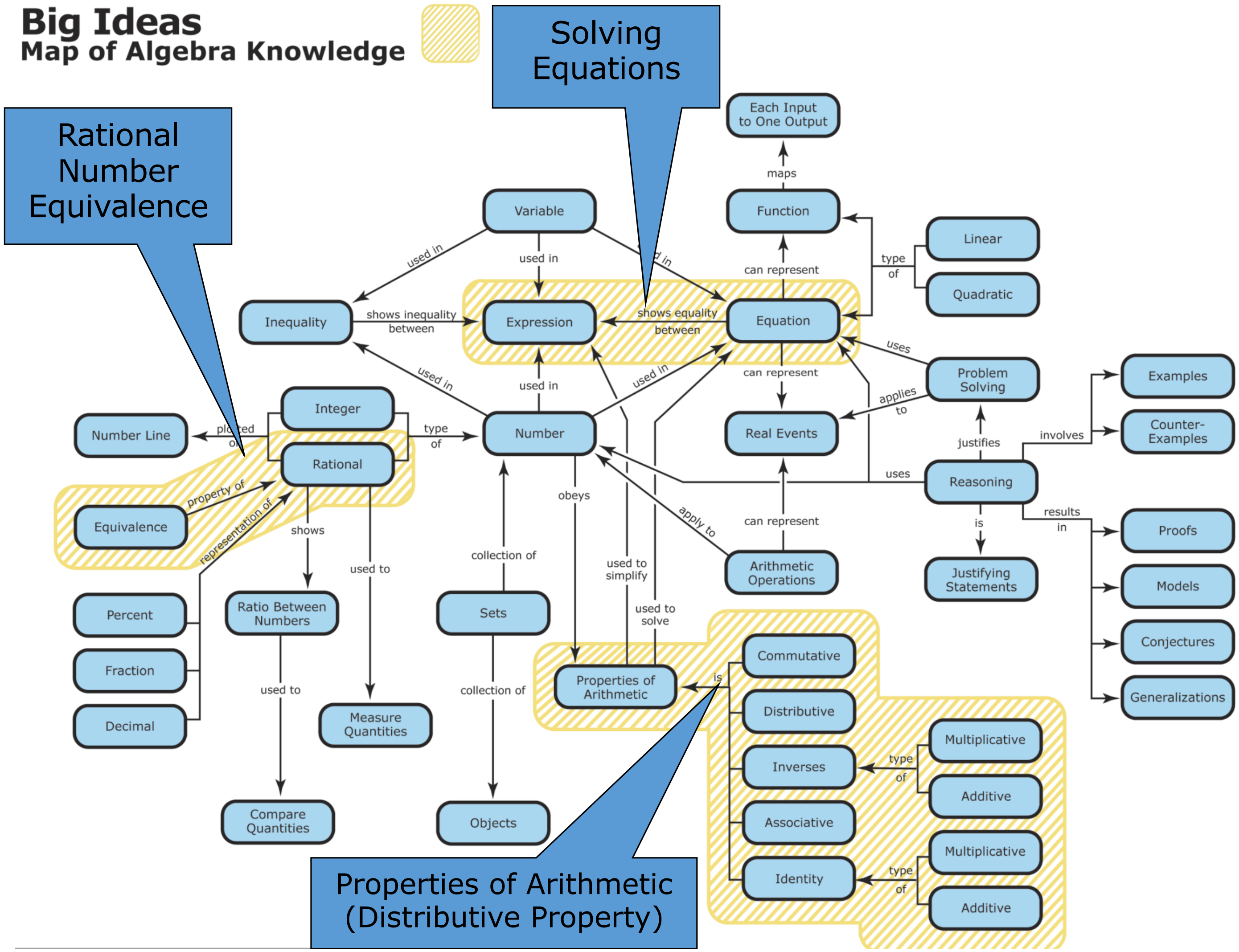
ONTOLOGY/BIG IDEAS MAP

- Expert-based representation of a domain
- Illustrates the connections between concepts
- Provides general description of domain—conceptual overview and big picture
- Explicit and transparent
  - You know what you're getting (for better or worse) and you can explain why things happen the way they do
- Graphical representation makes interactions and connections more obvious

Helps us choose powerful principles or big ideas.

These principles induce schema that support growth and transfer.

Big Ideas
Map of Algebra Knowledge



Professional Development focused on FA practice + content

The challenge to formative assessment?

Step 1 is easy(ish!)

Collect some assessment information.

The challenge to formative assessment?

Step 1 is easy (ish!)

Collect some assessment information.

Step 2 is hard.

How do you alter your instruction in the face of assessment information?

- Teacher knowledge can be a hurdle.
  - Pedagogical content knowledge as well as subject area knowledge.
  - Not sufficiently deep to teach or assess mathematics effectively.
- More time and energy is required to use assessments in a formative way.

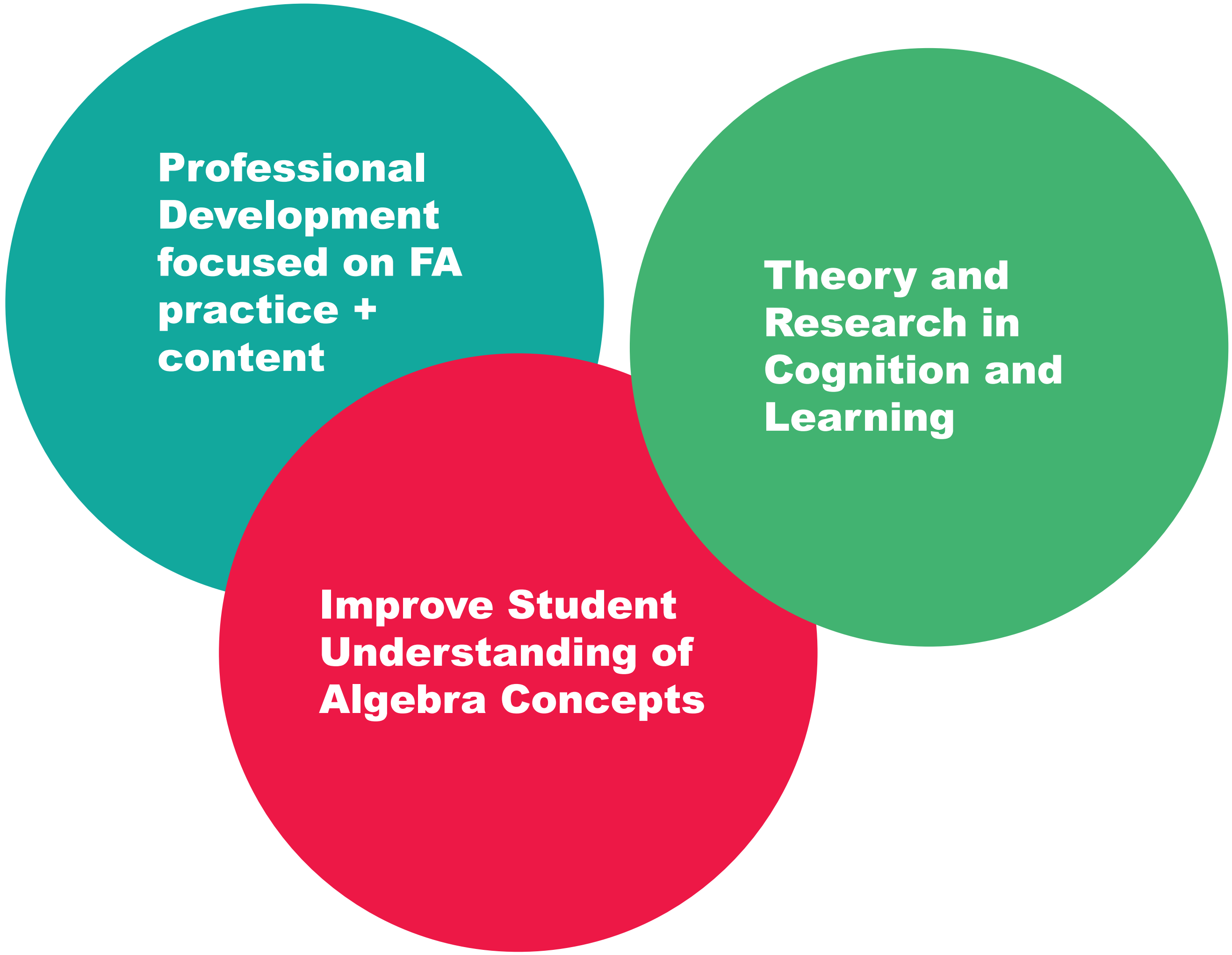
Formative Assessment Practice

Developing the **content knowledge** of teachers, especially content they will actually be teaching.

Developing a **community** of teacher-learners that can **actively share** teaching strategies and plan for classroom integration of these strategies.

Allowing teachers to **examine student work** and explore how student thinking (both accurate and inaccurate) develops.

High-Quality PD


Professional Development focused on FA practice + content
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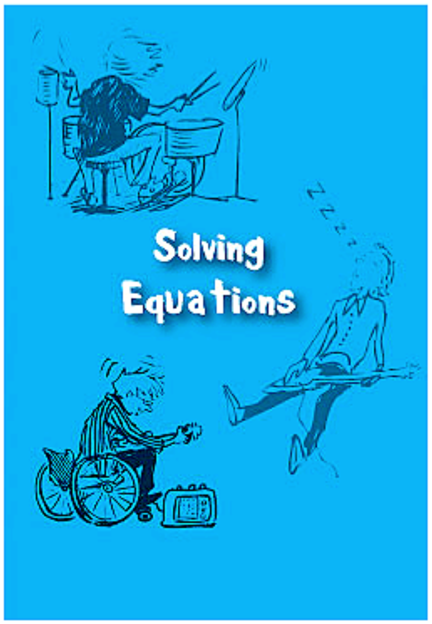
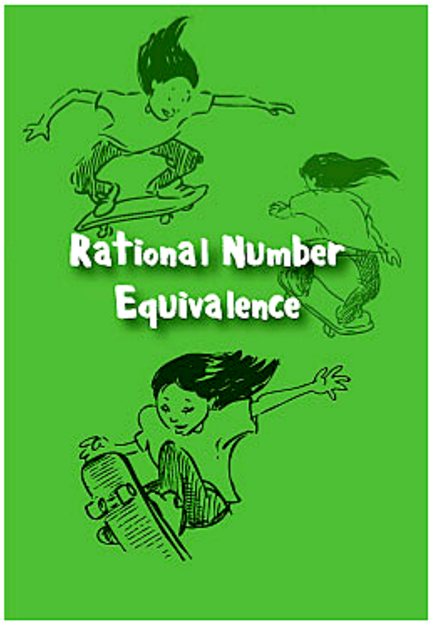
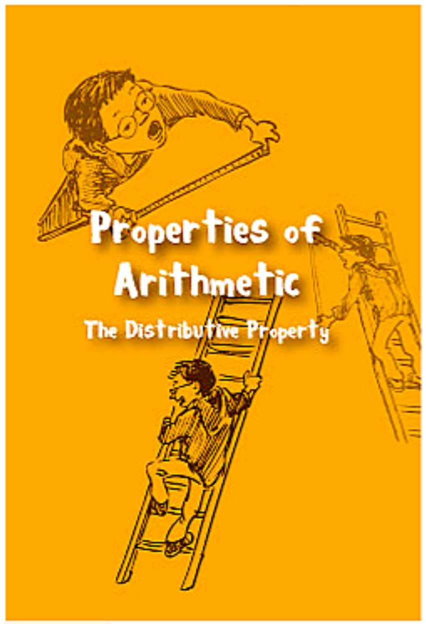
Improve Student Understanding of Algebra Concepts

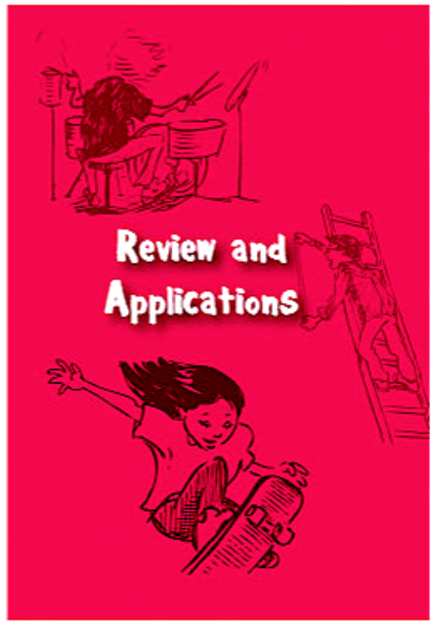
PowerSource

using assessment to improve student learning









PowerSource

Improve Student Understanding of Algebra Concepts

+

Theory and Research in Cognition and Learning

+

Formative Assessment Practice

Brief learning-based assessments to help students develop fundamental skills and knowledge

Focused on big ideas and schemas

Highly focused instruc tools to help those have not master ideas.

Based on learning resea and misconcepti

Goal is to develop assessment tools which are **sensitive to instruction** and which produce data teachers can use to make more **informed and effective** instructional decisions.

We chose three big ideas on which to focus our assessments:

These concepts were chosen as they satisfied the following criteria:

- Important to later mastery of algebra
- Significant place in math standards across grades 6-8
- Areas where students tend to have difficulties

Four modules, each comprised of three- day sequence of repeated assessments and instructional feedback/differentiated activity.

Within each content area we designed a series of short formative assessment measures (*Checks for Understanding*) to help teachers:

- assess their students' understanding of basic mathematical principles
- elicit student thinking and misconceptions about those big ideas.
- Connect to instruction and provide feedback to support deeper understanding.

Working with expert teachers from one of our participating districts, we developed four teacher handbooks—each closely aligned with one of the four content domains

❹ Here's how a student solved this problem $\frac{2}{3} = \frac{1}{\boxed{}}$.

$$\frac{2}{3} \bullet \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{\boxed{\frac{3}{2}}}$$

a) Is it correct to multiply $\frac{2}{3}$ by $\frac{\frac{1}{2}}{\frac{1}{2}}$? ______________ RN-EX-6 a

Explain your answer.

b) Is $\frac{2}{3}$ really equivalent to $\frac{1}{\frac{3}{2}}$? ________________

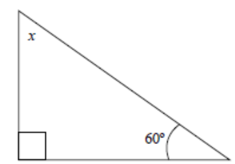
Explain your answer.

An extended response question to see if students can transfer the ideas about the multiplicative identity to a novel situation.

❹ The table below shows how a student solved the problem x + 2 = 20. Explain what the student did in step 2 and ***why*** she did it. Be sure to use some mathematical rule or principle in your explanation.

	Problem Solving Step	**Explanation**
1	x + 2 = 20	Wrote the problem
2	x + 2 – 2 = 20 – 2	$\boxed{}$
3	x + 0 = 18	Subtracted: 2 – 2 = 0, and 20 – 2 = 18
4	x = 18	Added: x + 0 = x. Adding 0 doesn't change the number.

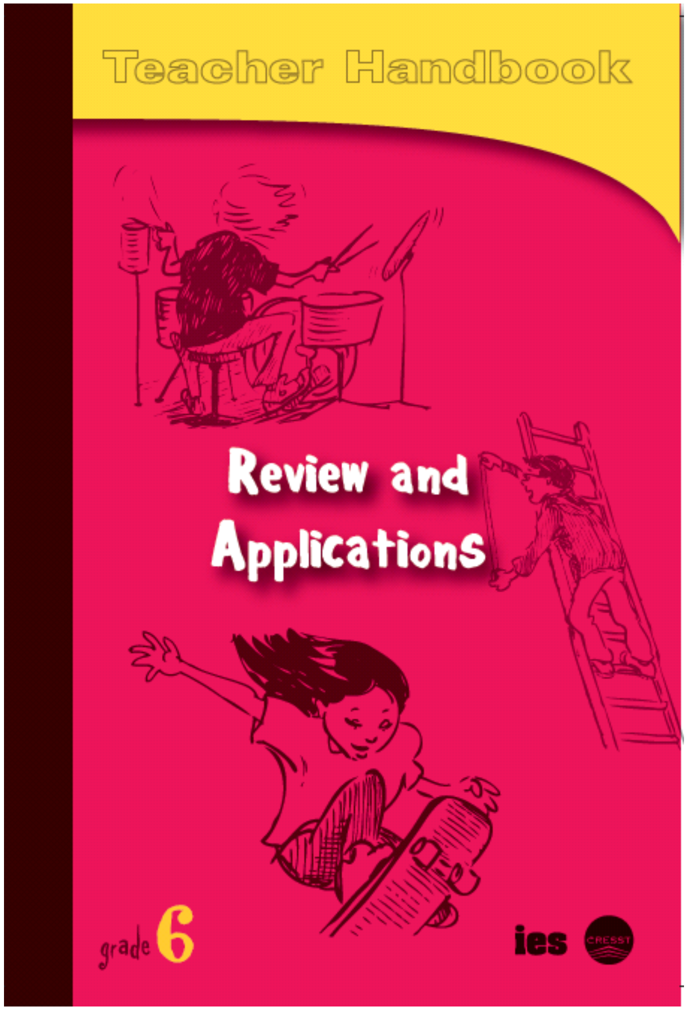
❸ A student was asked to solve for the value of x in the diagram below.



This is how the student set up the problem to find the missing angle (x). Can you fill in the missing numbers in step 2, 3, and 4?

	Problem Solving Step
1	$90 + 60 + x = 180$
2	$\boxed{} + x = 180$
3	$150 - 150 + x = 180 - \boxed{}$
4	$x = \boxed{}$

SE-PW-3 a
SE-PW-3 b
SE-PW-3 c



✓ *Handbooks to help teachers use the assessments in a formative way*

✓ *Guidance on interpreting and using information from the assessments*

✓ *Resources for teachers to help students who are having difficulty with the principles*

✓ *Suggested activities; maps, models and explanations of big ideas; worked examples to teach problem solving*

Teacher Handbook

Introducing the idea of the multiplicative identity

Lesson 1

say — You can write a fraction to show the ratio between any two lengths. Let's say you have a line segment that's 3 units long and another one that's 6 units long, like this:

show ➤➤➤➤➤➤➤➤ **Overhead 3**

say — What fraction can show the ratio between these two lengths? Right, 3 over 6. Can you name any other fractions?

Write and leave all the fractions on the board. If no one says a fraction with the same number over itself, write:

write — $$\frac{4}{4}$$

say — What does 4 over 4 equal?

After eliciting student responses, write:

write — $$\frac{4}{4} = 1$$

say — Any non-zero number written over itself like this equals 1. Why is this?

Teacher Handbook

Helping teachers understand and use the results of the assessments

Use the following chart to record the number of incorrect responses to each problem in Assessment 1:

	Problem	Incorrect Tally	Total
1	$75 \cdot 1 = \square$	||	2
2	Name and explain the property that you used to find the answer to question 1.	卌 ||||	9
3	$46 \cdot \frac{2}{2} = \square$	卌 |	6
4	$46 \cdot \frac{137}{137} = \square$	卌 卌	10
5	Explain why the following three fractions are equivalent: $\frac{2}{2}, \frac{100}{100}, \frac{1232}{1232}$.	卌 卌 |	11
6a	Explain why the following three fractions are equivalent to $\frac{2}{3}$.	卌 ||	7
6b	Explain how you found these fractions.	卌 |||	8

Similar but Different Contexts

Lead discussion about the fact that any non-zero number over itself equals 1.

Then ask students for other examples of fractions equal to 1. Draw a large 1 around the fractions.

draw

$\frac{11}{11}$ $\frac{5}{5}$ $\frac{3}{3}$

write

$\frac{11,167}{11,167}$

say

What about 11,167 over 11,167?
Does this fraction equal 1?

Then give students some fractions with only a numerator or denominator and ask how to make these fractions equal to 1. For example:

write

$\frac{}{7}$ $\frac{22}{}$ $\frac{47}{}$

say

So now you know what a rational number is. You know that fractions are rational numbers and you know how to find fractions that are equal to 1.

❸ Here's how a student solved this problem $\frac{2}{3} = \frac{1}{\boxed{}}$.

$$\frac{2}{3} \times \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{\boxed{\frac{3}{2}}}$$

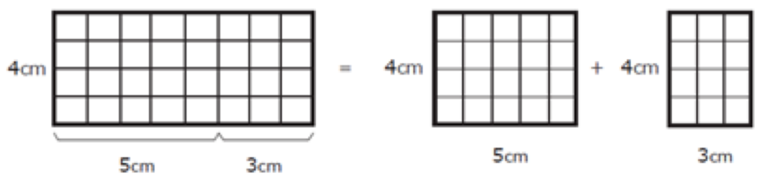
a) Is it correct to multiply $\frac{2}{3}$ by $\frac{\frac{1}{2}}{\frac{1}{2}}$? __________________

b) Is $\frac{2}{3}$ really equivalent to $\frac{1}{\frac{3}{2}}$? __________________

Explain your answer.

Different Representations

❼ A student drew these two diagrams to show how the distributive property works. Write a number sentence that goes with these diagrams. Be sure your number sentence shows the distributive property.



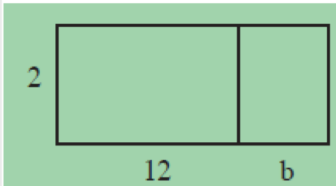
Complete the number sentence below.

PA-EX-11ab

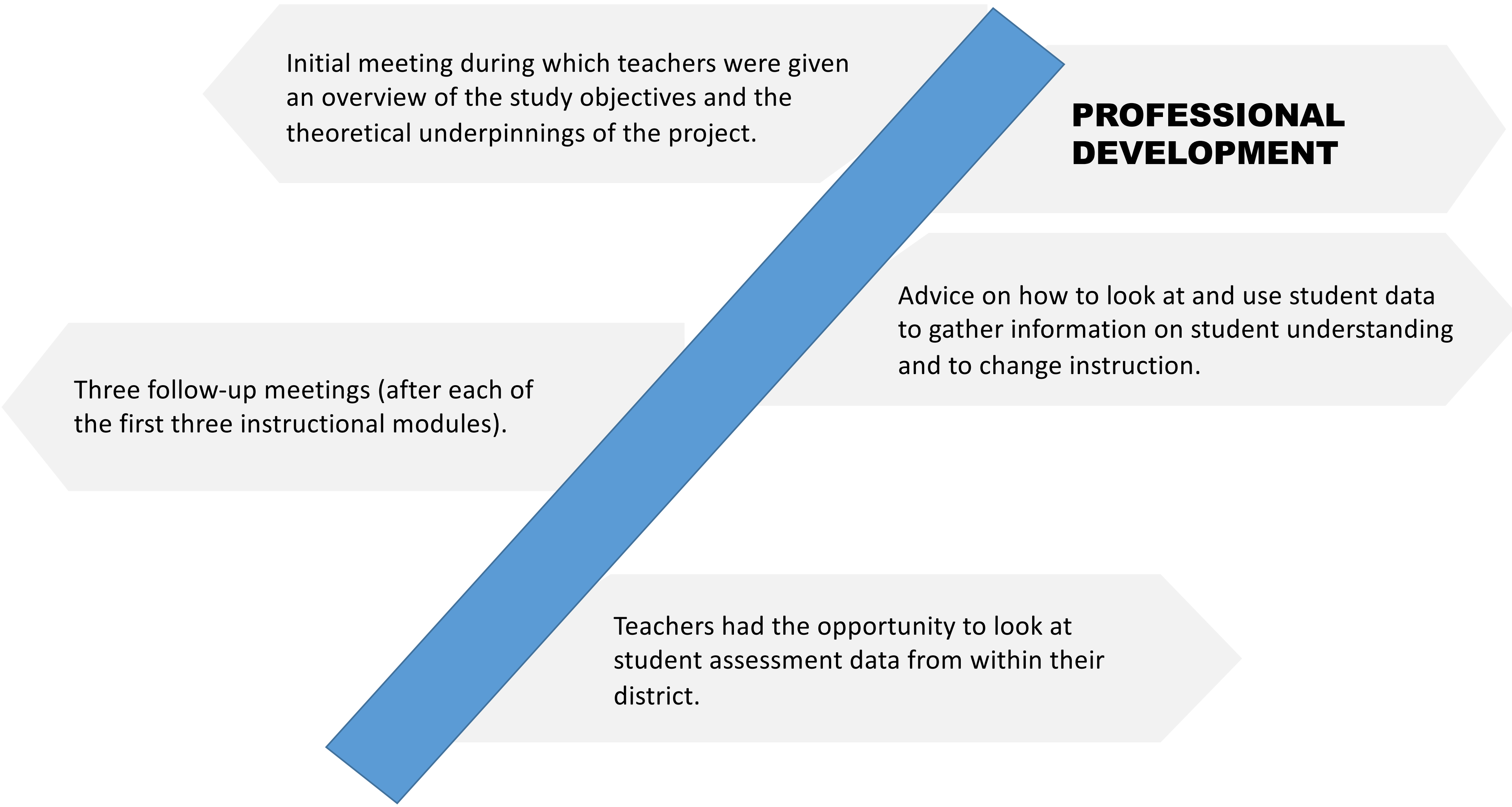
❺ $3\,(x + 5) = (\boxed{} \bullet x) + (3 \bullet 5)$

Using what you know about mathematical principles, explain why your answer is correct.

$$a(b + c) = (a \bullet b) + (a \bullet c) \quad \text{or}$$
$$a(b - c) = (a \bullet b) - (a \bullet c)$$



Besides counting, how can we find the number of dots on the board? We could do it two ways. We could add the 3 and the 4 across the top to get the number of columns in this diagram.


PROFESSIONAL DEVELOPMENT
Initial meeting during which teachers were given an overview of the study objectives and the theoretical underpinnings of the project.
Advice on how to look at and use student data to gather information on student understanding and to change instruction.
Three follow-up meetings (after each of the first three instructional modules).
Teachers had the opportunity to look at student assessment data from within their district.

a) Is it correct to multiply $\frac{2}{3}$ by $\frac{\frac{1}{2}}{\frac{1}{2}}$? ____________

Explain your answer.

Yes.

Because if its $\frac{\frac{1}{2}}{\frac{1}{2}}$, that is just 1.

Student understands one of the big ideas..

b) Is $\frac{2}{3}$ really equivalent to $\frac{1}{\frac{3}{2}}$? ____________

Explain your answer.

No.

Because if > 0, it would be $\frac{1}{1\,{}^{1}/_{2}}$.

..but understanding is not complete

6^{th} Grade Math

❶ $75 \cdot 1 = \square$

❷ a) Name the property you used to find the answer to question 1.

b) Explain this property.

Students can often use algorithms and solve problems, but can't always explain what they're doing.

❸ $46 \cdot \frac{2}{2} = \square$

❹ $46 \cdot \frac{137}{137} = \square$

❺ Explain why the following three fractions are equivalent.

$\frac{2}{2}$ $\frac{25}{25}$ $\frac{1143}{1143}$

Our Study

Random assignment implementation study of our formative assessment strategy.

- Teachers were randomly assigned to either treatment or control conditions.
- The treatment group students received instruction and *Checks for Understanding* on the four domains.
- Comparison group of students who received their regular instruction.

Sample (Grade 6)

District	Students (N)	Teachers (N)	Schools (N)	Design
AZ_1	275	7	3	B/S
CA_1	702	17	3	W/S
CA_2	390	9	3	B/S,W/S
CA_3	107	6	4	B/S
CA_4	867	30	11	B/S
CA_5	26	1	1	B/S

Based on district needs and configurations.

[redacted]

Student characteristic	AZ district 1	CA district 1	CA district 2	CA district 3	CA district 4	CA district 5	CA district 6
Asian	0%	6%	4%	1%	6%	12%	3%
Black	10%	3%	7%	1%	6%	13%	3%
Hispanic	33%	36%	41%	24%	70%	32%	76%
White or other	57%	55%	52%	74%	16%	13%	18%
EL	12%	12%	14%	10%	22%	26%	20%
Below proficient in mathematics, 2007	24%	50%	50%	49%	55%	53%	49%

Research Questions

What is the impact of the use of the POWERSOURCE© assessment strategy on student learning as measured by performance on assessments of key mathematical ideas?

Is there an impact of the number of years a teacher has been involved in POWERSOURCE© on student learning?

Does the impact of the intervention differ by variation of treatment design across different participating districts?

Is there an impact of the number of years a teacher has been involved in POWERSOURCE© on teacher knowledge and practice?

TIMELINE

Year 1

Pilot testing and development of materials

Year 2

Field testing assessments along with associated PD and instructional resources

Year 3

Grade 6 implementation

Ongoing pilot testing and development of additional materials

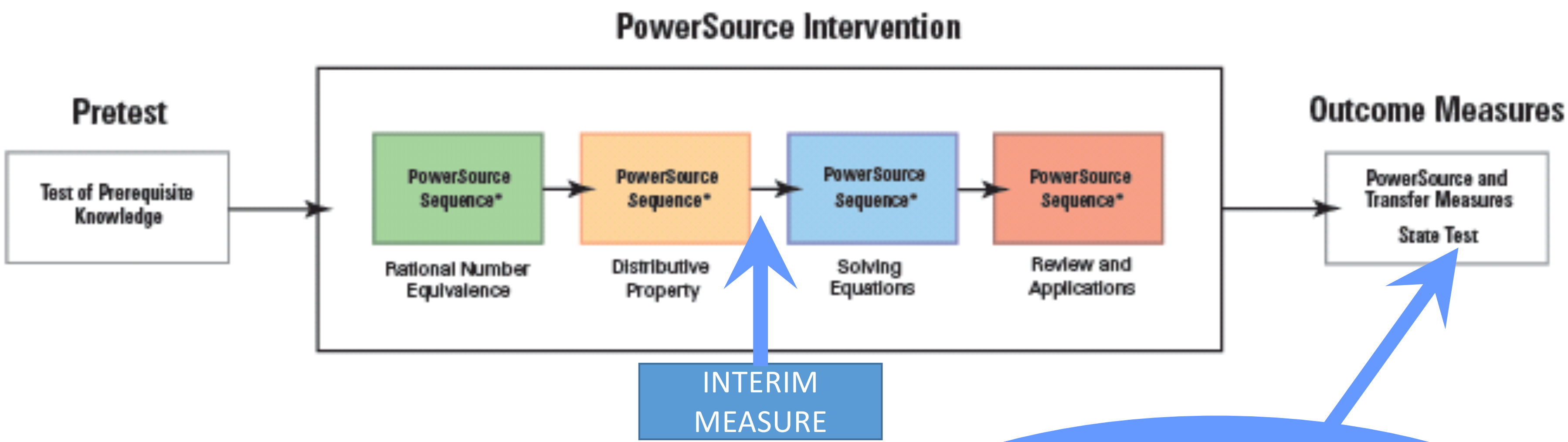
Year 4

Grade 6 and Grade 7 implementation

Ongoing pilot testing and development of additional materials

Year 5

Grade 6 , Grade 7 , and Grade 8 implementation



Transfer measure included items from TIMMS, NAEP, PISA, Key Stage 3, & local benchmark-test items.
Items were selected based on their relevance to the study domains and their appropriateness for a transfer task.

Data Analysis: Grade 6

We conducted three HLM analyses:

1. **Interim measure as the outcome→ pretest score as the covariate.**
2. **Transfer measure as the outcome → interim measure as the covariate.**
3. **Transfer measure as the outcome → pretest score as the covariate**

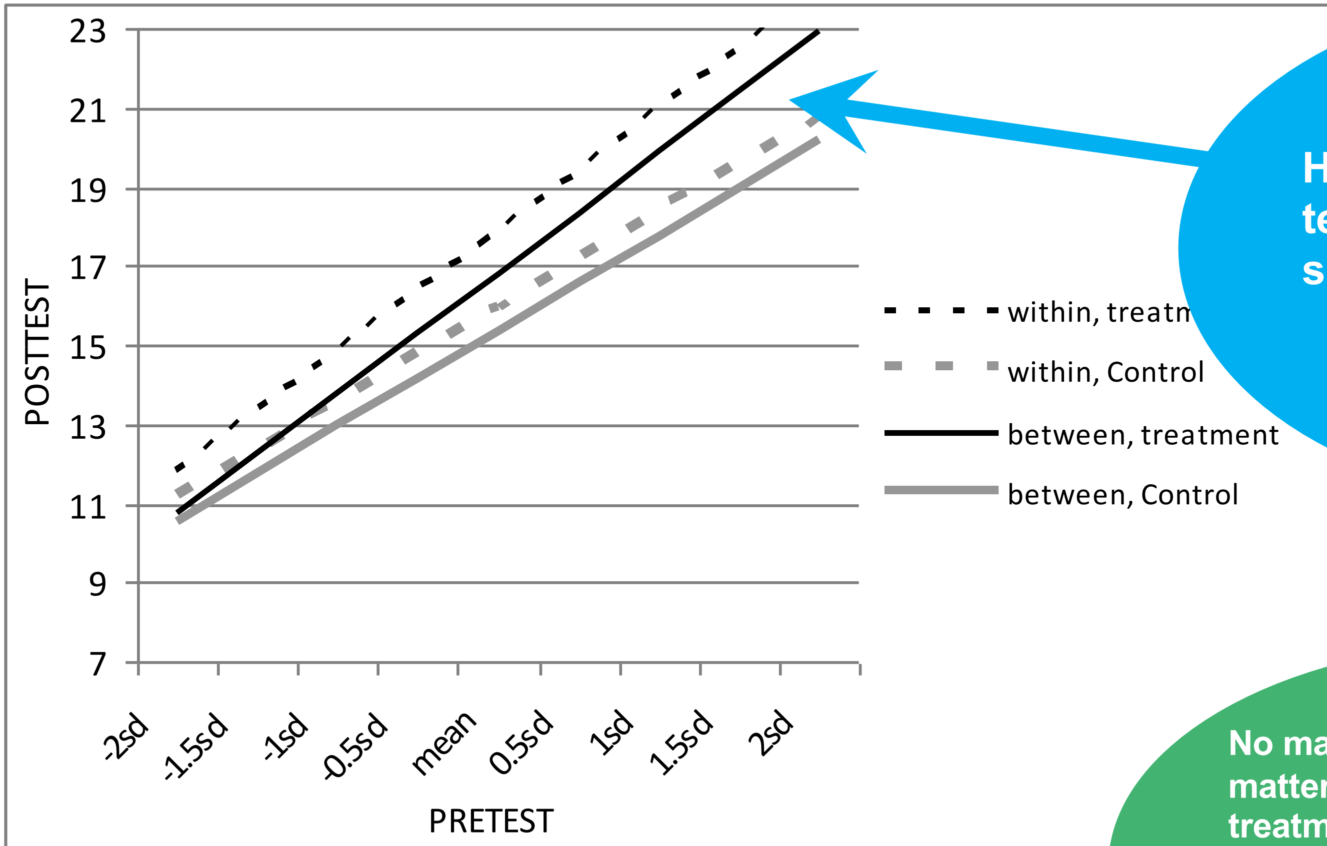
Findings: Grade 6

1

TRANSFER MEASURE AS OUTCOME:
Results indicated a significant **main effect of treatment** with the estimated coefficient equal to 1.95.
The **pretest main effect was also significant** (estimate = 1.05, $p < 0.001$).

Higher pretest mean scores tended to have higher mean scores on the transfer measure.

Students in the treatment group outperformed students in the control group



Higher pretest mean scores tended to have higher mean scores on the transfer measure.

No main effect of design, it did not matter—in terms of magnitude of the treatment effect, which type of design we used.

IX between treatment group and pretest score ns.

Findings: Grade 6

2

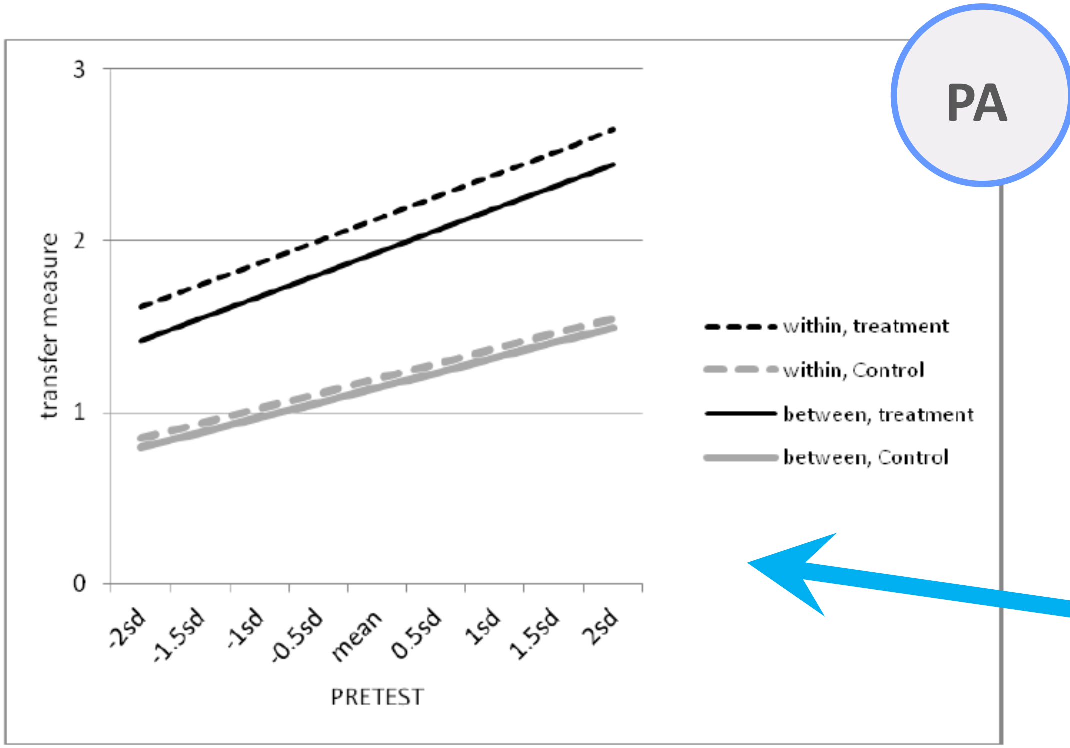
TRANSFER MEASURE SUBSCORES

PA: Statistically significant treatment effect (estimate= 0.93, p-value < 0.0001) and the pretest mean effect was also significant (estimate=0.52, p-value <0.0001).

RNE: There was a significant main effect of treatment (estimate= 0.70, p-value = 0.01). The pretest mean effect was also significant (estimate=1.30, p-value<0.0001)

For properties of arithmetic and rational number equivalence,
ts significantly outperformed students in

Item analyses indicated that PA and RNE items were more difficult for students that items focused on the other domains.



Significant POWERSOURCE intervention effects on students' 6th grade transfer measure PA sub-scores after controlling for students' pretest scores.

We found similar results for the eight items associated with the RNE domain.

Findings: Grade 6

3

INTERIM MEASURE

When the total interim score was used as the outcome, the HLM analysis revealed a significant **main effect of treatment** (estimate = 1.62, p = 0.00),

Students in the treatment group outperformed students in the control group on a measure of items related to PA and RNE.

PowerSource

- ✓ The POWERSOURCE intervention required **only twelve class periods** of classroom implementation (instruction and assessment), with an additional **nine hours of professional development** for teachers.
- ✓ The instructional modules, assessments and professional development sessions **complemented existing curricula** and fit around what districts and schools already have in place.
- ✓ Our intent was to implement an intervention that **would augment**—not replace—mathematics instruction already in place in the districts.

Time for the intervention had to be found within sometimes tight curriculum frameworks and timelines.

A substantial part of our research focused on exploring the types and frequency of assessments that will be feasible to implement and most beneficial to teachers and students. Helping teachers to:

- **understand mathematical concepts more deeply**
- **monitor learning of key ideas**
- **figure out the best strategies to improve students' understanding**

Findings and Discussion

- For most of the study years, interaction effects were observed: the intervention showed **benefits for higher-performing students** but not for lower-performing students.

- In most of the analyses, we found those students who began with a **higher pretest score performed better** on the outcome measures than those who had lower pretest scores.

- The final year results showed **a main effect of treatment for 6th grade students**.

  - Study implementation started with sixth grade teachers, and thus these teachers had more implementation experience (potentially 3 years) than those at grades 7 and 8 (2 and 1 year respectively).

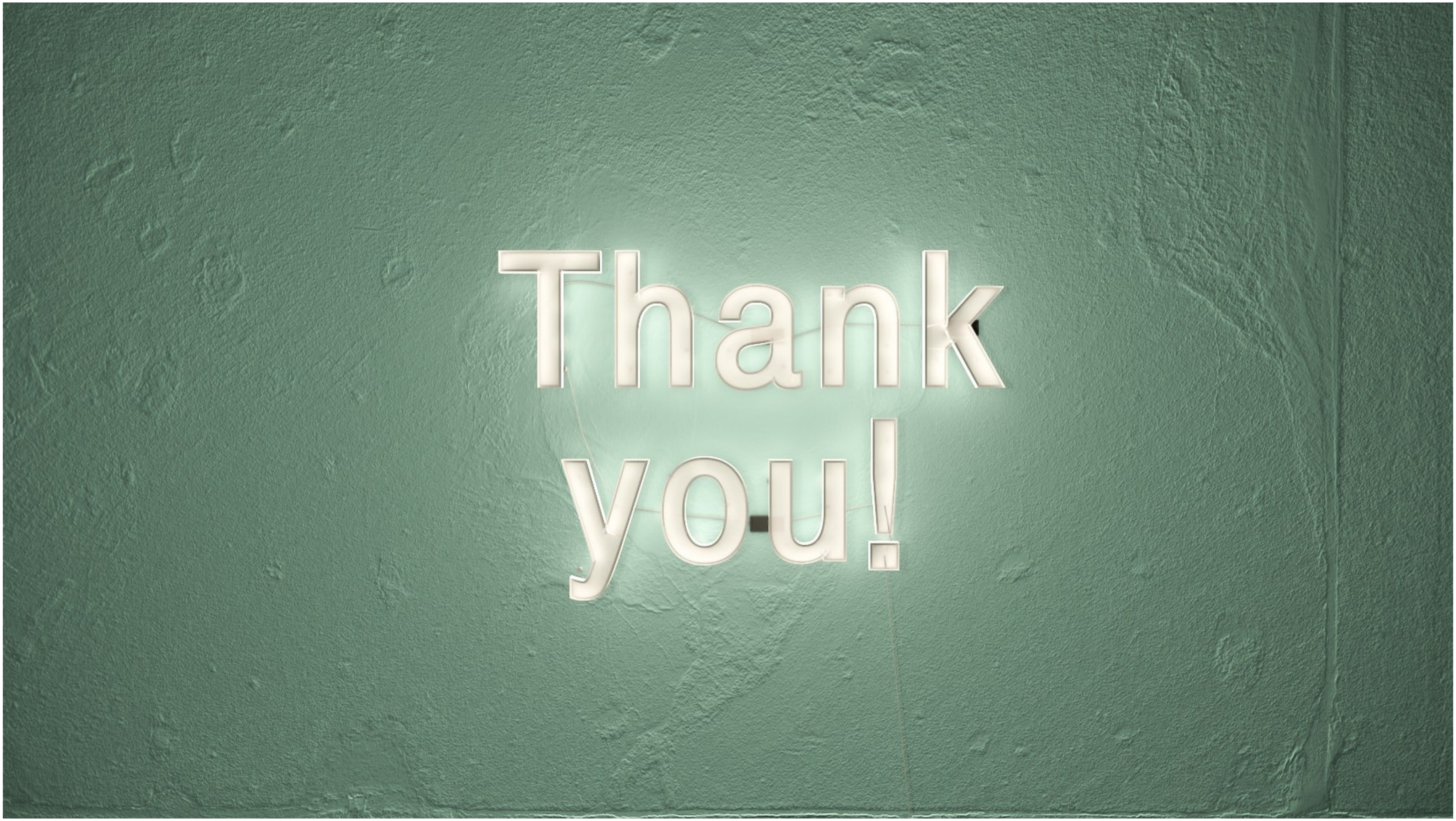
- On the interim outcome measure (which contains items from the most difficult domains—PA and RNE) the **number of years teachers had participated was an important factor on students learning.**

- When main effects or interaction effects were not found for total transfer measure score, we did observe significant **main effects for subscores related to RNE and PA.**

Findings and Discussion

Teacher Measures (pre and post)

- Knowledge map task
  - Overall the treatment teachers in 6th and 7th grades made significant gains in their ability to organize concepts on a knowledge map activity
    - After one year of PD, the **6th grade treatment teachers** were **better able to organize concepts** in the knowledge map task than were their peers in the control group once pre-PD ability was controlled for.
    - After two years of PD, 6th grade teachers were better able to **connect problems with the associated concepts necessary to solve those problems**, relative to the control and after ability prior to PD was controlled for.
    - 7th grade treatment teachers made significant gains in their ability to organize concepts. However, these gains were **only evident after two years** of professional development.


Thank
you!