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One-to-One Play Promotes Numerical Equivalence Concepts

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Young children spontaneously engage in a variety of one-to-one correspondence activities during play. The present study tested whether one of these activities—pairing objects with containers—supported the development of numerical equivalence judgments. Three-year-olds were given sets of toys to take home. In one condition, the toys were accompanied by a container that facilitated one-to-one matching (e.g., wiffle balls and a muffin tin). In the other condition, the same toys were accompanied by a numerically equivalent set of loose objects (e.g., wiffle balls and plastic frogs). At test, children who had played with the containers showed significant improvement on a challenging number-matching task (i.e., cross mapping). In contrast, children who had played with the loose sets of objects continued to perform at chance.

One-to-one correspondence is at the core of number concepts. The one-to-one mapping between number words and items to be counted is what makes counting meaningful. One-to-one correspondence also provides a way to determine equivalence and ordinality without counting. That is, if all the individual items in two groups can be matched up, one to one, then the groups are equal in number. If the items cannot be matched up, then the group with leftover items is larger.

Although the conceptual relations between one-to-one correspondence and number are clear, the developmental relations are less so. Piaget

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(1941) believed that young children lack a fundamental understanding of one-to-one correspondence because they perform so poorly on the number conservation task. In this task, children watched as an experimenter laid out two rows of counters. At first, the counters in each row were directly across from one another, and under this condition, children readily judged the two rows to be equal in number. However, when the experimenter next transformed one of the rows by spreading it out or pushing the items closer together, children often judged the longer row to have more counters in it. This error usually persists until 6 or 7 years of age. Piaget argued that the error would not occur if children understood the logical implications of one-to-one correspondence because they could either recall the previous correspondence of the identical rows or recheck the correspondence between items in the transformed rows.

Subsequent research indicated that children recognize numerical equivalence much earlier than Piaget thought (Gelman, 1969; Gelman & Tucker, 1975; Huttenlocher, Jordan, & Levine, 1994; Mix, 1999a, 1999b; Mix, Huttenlocher, & Levine, 1996; Siegel, 1971). For example, when children are shown a set of disks and then asked to choose a card showing an equivalent set of dots, children as young as 3 years old perform significantly above chance (Mix, 1999a, 1999b; Mix et al., 1996). Of course, we cannot know whether such judgments are based on one-to-one correspondence. Preschoolers can recognize and name the numerosity of small sets without counting (Fuson, 1988; Wagner & Walters, 1982; Wynn, 1990) and therefore could use cardinality to match sets. Still, nothing would prevent the use of one-to-one correspondence in such tasks. Indeed, most accounts of early number processing assume children do (e.g., Feigenson & Carey, 2003; Feigenson, Carey, & Hauser, 2002; Gallistel & Gelman, 1992; Huttenlocher et al., 1994; Spelke, 2003).

Although children begin to recognize numerical equivalence by 3 years of age, they do not immediately do so for every comparison. They first detect equivalence for identical or nearly identical sets (e.g., two black disks = two two black disks) and only later do so for more disparate sets (e.g., two lion figures = two black dots; two drumbeats = two black dots, etc.; Huttenlocher et al., 1994; Mix, 1999b; Mix et al., 1996). The most difficult number-matching condition pits number against object identity (Mix, 2008a). For example, given a standard set of two cars, children would choose between a number match, such as two dogs, and an object match, such as three cars. Identifying the number match requires children to ignore all the other possible commonalities between sets. Cross-mapping tasks such as this, which pit a specific relation against other object properties, are typically difficult for preschool children (e.g., Rattermann, Gentner, & DeLoache, 1989). The number cross-mapping task was no exception. Children performed at chance until 4 years of age and continued to perform significantly worse than on

more literal comparison conditions (e.g., two disks = two dots) until 5 years of age (Mix, 2008a).

What developmental mechanisms might underlie the progressive abstraction of numerical equivalence? We can look to the literature on category development for potential mechanisms because matching numerically equivalent sets is essentially a categorization task. Just as children might group together cars, or dogs, or cookies, so might they group sets of two, sets of three, and so forth. All such groupings are based on recognition of similarity; in this case of numerical equivalence, groupings are based on numerical similarity. So, to uncover the mechanisms that lead to numerical abstraction, we must begin by considering how children would discover numerical similarity.

Research indicates that children discover new dimensions of similarity by making comparisons based on already-known points of alignment (Christie & Gentner, 2010; Gentner, 2003; Gentner, Loewenstein, & Thompson, 2003; Gentner & Namy, 1999; Namy & Gentner, 2002; Smith, 1989, 1993; Waxman & Klibanoff, 2000). Initial comparisons are supported when items have many features in common because this increases the chances that children will detect at least one known commonality and begin to align the items. Once this alignment has begun, children will seek out additional points of comparison, thereby discovering new dimensions of similarity. In this way, comparisons build new conceptual structures, and these developing structures, in turn, lead to more abstract and detailed comparisons.

Numerical equivalence concepts could be built this way. Imagine children encounter two sets of objects, having no awareness of numerical equivalence as a dimension of similarity. If the sets share other commonalities, children may be drawn to compare them for other reasons. Perhaps the items are the same color, shape, or texture. The two sets also might be configured in similar patterns or aligned with each other spatially. They might have the same name ("ducks" or "red" or "four"). The more ways these two sets are similar, the more likely children would be to compare them. By aligning the sets for whatever nonnumerical reason, children might also align them numerically (via either counting or one-to-one correspondence), thereby discovering numerosity as a point of alignment, separate from other commonalities.

Existing research on numerical equivalence judgments is consistent with this scenario. As noted above, children first recognize numerical equivalence when there are many shared features, both between the sets as wholes and the objects within them (Mix, 1999a, 1999b, 2008a, 2008b; Mix et al., 1996). This suggests that children need a great deal of perceptual support to begin comparing sets. They apparently do not see number as a separate dimension of similarity early on, but they may well discover this relation after structurally aligning highly similar object sets for other reasons. Previous research also indicates that children perform better on numerical comparisons when

they know the meanings of at least a few number words. This provides indirect evidence that shared labels promote numerical comparisons because children who can label sets of one, two, or three are at least capable of assigning shared labels to equivalent sets (Mix, 1999a, 2008a, 2008b).

Although intriguing, these findings fall short of directly demonstrating that comparisons between sets lead to numerical abstraction. Furthermore, there are reasons to wonder whether high similarity and shared labels are the best way to highlight numerical similarity. Although number is like other categories, it also has distinctive properties. Perhaps more than any other category, number is psychologically imposed. It is more difficult to recognize the “twonesses” in a pile of toys than it is to recognize the cars, for example. To see number, one must first decide to group something, then group something else, then determine the numerosity of these groups, and recognize these numerosities as similar. Moreover, the internal structure for numerical equivalence is very specific. It is based on one-to-one correspondence—not other dimensions of quantity, such as overall surface area or mass, or other features of the sets, such as shape. Thus, a particularly potent form of input for numerical equivalence might be pairing items one to one.

In theory, there are many contexts that could elicit one-to-one pairings between sets. One could be high surface similarity between individual items across but not within sets (Gentner, Rattermann, Markman, & Kotovsky, 1995; Paik & Mix, 2008; Tversky, 1977). For example, a rich, heterogeneous set that includes a doll, a car, and a cup is likely to invite one-to-one matching with an identical set because there are many shared commonalities across items and sets, as well as enough distinctiveness to highlight potential pairs. Similarly, functional or thematic pairings might elicit one-to-one matching (e.g., Greenfield & Scott, 1986; Markman & Hutchinson, 1984). For example, a set of mommy animals might elicit matches to a parallel set of baby animals. Or spoons could be placed in bowls. Indeed, Sugarman (1981) observed that 2- and 3-year-olds sometimes touched such pairings sequentially, although these instances were much less frequent than sequential touching within category (e.g., touch all the bowls, then all the spoons).

In the present study, we focused on another potentially powerful context: fitting objects into containers with slots. One reason for this choice is that fitting objects into containers is a naturally occurring play activity in toddlerhood (Mix, 2002, 2009). In an 18-month diary study of her son, Spencer, Mix observed many instances of spontaneous, one-to-one correspondence; however, these rarely involved matching one set of objects to another—even those with thematic or featural overlap. Instead, he tended to match objects to people as he distributed toys and food, and somewhat later, placed objects in containers. Second, objects-with-slots activities emerged just before numerical equivalence concepts and may have special conceptual status. Both early

types of correspondence (i.e., distributed objects and objects with slots) could be constructed from local matches and thus might be easier to make than loose object matches. However, objects-with-slots matches may have been more informative because they resulted in sets that were easier to observe and compare. If children hand out cookies to a playgroup, the sets will be moving. One set will be eating the other. However, if children place eggs into an egg carton, both sets and their equivalence remain in full view, unlikely to change. Furthermore, empty slots seem to beg the question, "Where are the others?" Indeed, such questions were among Spencer's first explicit comments on equivalence, and these were elicited in play with objects and slots, just before he began to correctly match equivalent sets in a forced-choice task (Mix, 2002).

Additional support for this hypothesis comes from research on number conservation. Recall that children failed to recognize numerical equivalence for rows of disks when one row is longer than the other (Piaget, 1941). However, Piaget reported improved performance in another condition for which objects matched with slots. For example, he found that 6-year-olds conserved number when they were asked to compare a row of eggs to a row of egg cups, or one row of flowers to a row of vases (Piaget & Inhelder, 1978). Piaget did not consider this evidence of conservation because children continued to fail the original version of the task wherein perceptual supports are not provided (i.e., with disks). Still, he may have hit upon the mechanism by which children achieve this understanding. If objects with slots can support numerical inferences in an experimental task, then they have the potential to support numerical inferences in naturalistic play. Such comparisons may be an important source of input as children develop numerical equivalence concepts. Although Piaget did not find evidence of spontaneous transfer on the conservation task, children may need to experience many of these correspondences before numerical equivalence becomes a salient dimension for comparison in its own right. Thus, it is possible that repeated exposure to comparisons supported by object-with-slots correspondences would lead to such abstraction.

The present study is designed to test whether this is so. We provided toys that invited one-to-one pairing by way of placing objects in slots, and then we tested the effects of this exposure on a cross-mapping version of the triad-matching task. If our hypothesis is correct, performance should improve after experience with matching sets.

METHOD

Participants

Fifteen boys and 15 girls completed the experiment (mean age = 3;7; range = 3;1 to 4;0). An additional 7 children were excluded for performing

above chance on the pretest. Children were recruited through local preschools and day care centers. Participants came from a predominantly White, middle-class population and all spoke English as their primary language.

Materials and Conditions

Children were randomly assigned to one of two conditions: objects with slots ($n = 16$) and objects with objects ($n = 14$). In both conditions, children received three sets of toys to take home. These were distributed one set at a time in sequential 2-week rotations for a total of 6 weeks of exposure. The order of the particular toy sets was counterbalanced across children. Toys in the objects-with-slots condition consisted of one set of objects and a container with an equivalent number of openings, into which the objects could fit snugly (e.g., six wiffle balls and a muffin tin). Toys in the objects-with-objects condition were two homogeneous, numerically equivalent sets of loose objects (e.g., six wiffle balls and six plastic frogs; see Figure 1). A complete list of the stimulus objects used in the two conditions is provided in the Appendix.

Children in both conditions were tested with the cross-mapping version of the number-matching task used in previous research (Mix, 1999a, 1999b; 2008a, 2008b; Mix et al., 1996). This task was chosen because it requires children to isolate number from other dimensions of similarity and is sufficiently challenging for preschool children to reveal improvement from training. On each trial, a display with the target number of stickers (two, three, or four) was presented and left in full view while the child chose an equivalent display from among three choices. Both the target and choice displays were constructed from 5" \times 8", unlined, white index cards. Each



FIGURE 1 Examples of toys: balls in tin (objects with slots) and balls with frogs (objects with objects). (Color figure available online.)

choice card contained a homogeneous set of stickers, ranging in number from one to five, that contained richly detailed photographs of familiar items, such as animals, foods, vehicles, and flowers (see Figure 2). All of the stickers on the numerically equivalent card were different from the stickers on the target card. In contrast, one of the foil cards was an object-level match. It contained a different number of items than the target card, but the stickers on one card were a subset of the stickers on the other. The other foil differed from the target card in terms of both number and the particular stickers used. It was included to detect random guessing.

Note that the set sizes of the toy sets were larger than the set sizes in the cross-mapping task. Specifically, set sizes in the toy sets ranged from six to eight objects, whereas set sizes in the cross-mapping trials ranged from two to four. Because we were building on previous work using the triad-matching task, we used the same set sizes and materials. However, it is possible for preschool children to estimate the cardinality of small sets (i.e., two to four items) using either verbal count words or a nonverbal object-tracking system (e.g., Carey, 2001; Huttenlocher et al., 1994; Kahneman, Treisman, & Gibbs, 1992; Uller, Carey, Huntley-Fenner, & Klatt, 1999). Thus, large sets were used in the training materials to encourage children to match items one to one, rather than estimating cardinality.

The cross-mapping task consisted of 12 trials presented in one of two fixed random orders. The choice cards were constructed so that on one third of the trials, the numerically equivalent choice was either the largest in number, the smallest in number, or the middle value of the three choices. The stickers on the target and choice cards were arranged in lines. On half of the trials, the linear arrays on the choice cards were equated for length. On the other half, the arrays were equated for density. Children were prevented from matching the sets in terms of these variables because the linear arrays on the target

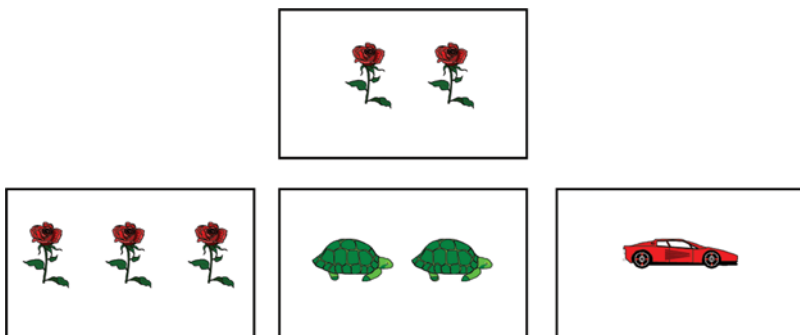


FIGURE 2 Sample triad in the number cross-mapping task. (Color figure available online.)

cards were always presented in the alternate format. That is, on trials for which the choice cards were matched for line length, the target set was presented as it would appear in a density-controlled pair. Similarly, when choice cards were matched for density, the target set was presented as it would appear in a line length-controlled pair. The position of the number match was counterbalanced across trials so that it appeared in all three positions equally often. To allow the experimenter to present all three choice cards simultaneously, the cards for each trial were attached to a 27.5" × 5" piece of black poster board using hook-and-loop tape (i.e., Velcro).

Procedure

Children completed the cross-mapping task at pretest and posttest. Cross-mapping trials began with the rows of choice displays placed facedown in front of the child. The cards with the target sets also were placed facedown between the experimenter and the choice displays. On each trial, a target display was turned over and left in full view of the child for a few seconds. Next, with the target set still in full view, the first set of choice cards was turned over to reveal the three arrays of stickers. Children indicated their choices by pointing. Each assessment, pretest and posttest, took approximately 20 minutes to complete. Children were tested in their preschool classrooms.

The task was introduced with a brief series of familiarization trials using target displays of one and two. The familiarization procedure was based on that used in previous work (i.e., Mix, 1999a, 1999b, 2008a, 2008b; Mix et al., 1996). First the experimenter said, "We're going to play a game. I'll show you how it goes." Then, she demonstrated the task by presenting a target set and pointing to the numerically equivalent choice card while saying, "See? This card goes with this card." The child was told, "Now it's your turn," and received 2 practice trials—1 with the target set just used in the demonstration and another with a different target set but the same choice cards. This sequence was then repeated with a different set of target and choice cards. During the practice trials, children were told whether or not their responses were correct. When children were correct, the experimenter said, "Right! That card goes with this card. Good job!" When children were incorrect, the experimenter said, "Nope, it's not that card. *This* card goes with this card," while pointing to the correct choice. Children were encouraged to point with the experimenter, and when they did, the experimenter said, "Right! *That's* the card that goes with this card. Good job!" No feedback was given during the 12 test trials. Only children who performed at chance or worse on the pretest continued their participation to the training phase.

During the training phase, children brought home three sets of toys from their assigned condition (see Appendix). The specific sets of toys were

rotated every 2 weeks in a random order, resulting in 6 weeks of exposure to the training materials for each child. Parents were instructed to make the toys continuously available to their children and to record the amount of time and the way children played with the toys, but they were not asked to initiate or participate in any particular activities with the toys. Our hypothesis was that the toys themselves would invite one-to-one matching, thus simulating the kind of information children might generate in their own spontaneous play. If we had instructed parents to perform certain activities with the objects-with-slots toys, it would have been unclear whether any resulting effect was due to the properties of the toys, the parents' instruction, or both. Moreover, if parents were given parallel instructions in the objects-with-objects condition, these actions would have increased the likelihood of significant effects in both conditions, further obscuring the potential impact of the toys themselves. Although we did not instruct them to perform specific activities with the toys or even to play with their children, we also did not ask them not to do so. Thus, parents in both conditions were free to play with their children as they liked, and it is possible this play included one-to-one pairings.

RESULTS

Figure 3 shows the mean scores of children in the objects-with-slots and objects-with-objects conditions on both the pretest and posttest. As expected based on previous research (and also because high-performing children were excluded), children in both conditions found the cross-mapping task difficult and performed poorly on the pretest—significantly below chance, in fact, due to the tendency to choose the object match over the number match (objects with slots: $M = 1.69$, $SD = 2.09$; $t(15) = -4.43$, $p < .05$; objects with objects: $M = 2.43$, $SD = 2.68$; $t(13) = -2.19$, $p < .05$). The scores of both groups improved on the posttest; however, only children in the objects-with-slots group performed above chance (objects with slots: $M = 5.81$, $SD = 3.06$; $t(15) = 2.37$, $p < .05$; objects with objects: $M = 4.00$, $SD = 3.11$; $t(13) = 0.00$, $p > .50$). Indeed, children's mean scores following objects-with-slots training approached the level of 4-year-olds in Mix's (2008a) cross-sectional experiment involving the same task (48% vs. 55%, respectively). A repeated-measures analysis of variance (ANOVA) confirmed that the interaction between test (pretest vs. posttest) and condition was significant, $F(1, 28) = 6.05$, $p < .05$. Specifically, there was greater improvement from pretest to posttest in the objects-with-slots toys, $t(15) = 6.46$, $p < .0001$, than in the objects-with-objects group, $t(13) = 1.88$, $p < .10$). Also, whereas the posttest scores for objects-with slots children

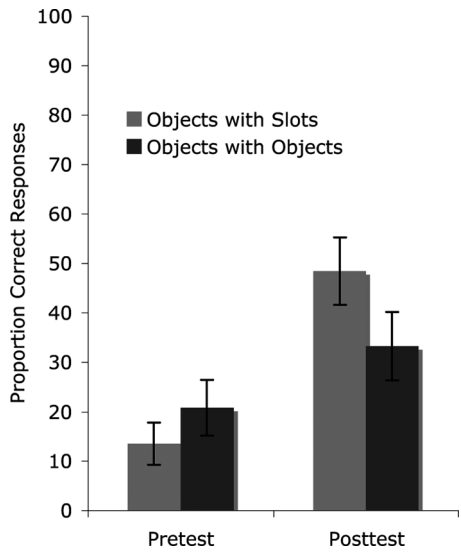


FIGURE 3 Mean cross-mapping scores (pretest and posttest) and standard error in the two training conditions.

were marginally higher than those for objects-with-objects children, $t(28) = 1.55$, $p = .065$, pretest performance was not significantly different between the two groups, $t(28) = 1.03$, $p = .15$). The ANOVA also yielded a significant main effect of test (pretest vs. posttest; $F(1) = 30.13$, $p < .001$), but there were no other significant main effects or interactions.

The same pattern was obtained when the test scores of individual children were considered. Children were divided into groups based on their cross-mapping scores. Scores of 3 to 5 correct were considered “at chance,” based on the binomial probability of answering correctly by guessing on a 12-item test with three choices per item. At pretest, most children in both conditions performed at or below chance (see Table 1). However, the distribution for

TABLE 1
Number of Children Performing Below, At, and Above Chance on the Cross-Mapping Task

	Objects with slots		Objects with objects	
	Pretest	Posttest	Pretest	Posttest
Below chance (0–1 correct)	9	2	6	4
At chance (2–5 correct)	6	4	6	5
Above chance (6 or more correct)	1	10	2	5

children in the objects-with-slots condition shifted significantly on the posttest, with many more performing above chance, $X^2(1, N = 16) = 7.47$, $p < .05$). This was not the case for objects-with-objects children, whose distribution of individual performance remained roughly the same, $X^2(1, N = 14) = 5.83$, $p > .20$.

Twenty-five parents (13 in the objects-with-slots condition and 12 in the objects-with-objects condition) returned logs in which they described the duration and types of play engaged by the experimental toys. The logs were coded for instances of one-to-one correspondence, including 1) aligning the objects with other objects, and 2) placing the objects in slots. Note that although the toys we provided lent themselves to one activity more than the other, it was possible for children to combine the experimental objects with other objects in their personal toy collections. Thus, children might align objects in the objects-with-slots condition and also put objects in slots in the objects-with-objects condition. However, this was rarely reported (see Table 2). Four of the logs were coded by two raters, and interrater correlations on all four were high ($> .90$).

Children in both conditions played with the experimental toys roughly the same amount of time (objects with slots: $M = 2.97$ minutes/day, $SD = 3.99$; objects with objects: $M = 3.05$ minutes/day, $SD = 2.53$; $t(24) = 0.097$, $p > .90$). Parents also reported roughly the same number of activities across conditions (see Table 1; $F(1, 24) = 0.093$; $p = .76$). However, children in the two groups differed significantly in the type and amount of one-to-one play. Not surprisingly, children placed objects in slots much more frequently in the objects-with-slots condition, $F(1, 24) = 18.19$, $p < .001$, and aligned objects more frequently in the objects-with-objects condition, $F(1, 24) = 7.01$, $p < .05$. However, the overall rate of one-to-one play was also higher in the objects-with-slots condition, $M_{\text{Slots}} = 0.40$, $SD = 0.31$; $M_{\text{Loose Objects}} = 0.09$; $SD = 0.12$; $F(1, 24) = 10.08$, $p < .005$. Thus, objects-with-slots toys invited significantly more one-to-one mappings.

Objects-with-slots parents frequently reported that children played with both sets, by repeatedly removing objects from their slots and replacing

TABLE 2
Parent Reports of Children's One-to-One Correspondence Play with Study Toys

	<i>Objects-slots</i>			<i>Objects-objects</i>		
	<i>Mean</i>	<i>SD</i>	<i>%</i>	<i>Mean</i>	<i>SD</i>	<i>%</i>
Total reported activities	12.47	6.21	100	11.45	10.67	100
Put objects in slots	4.00	3.10	39.64	0.00	0.00	0
Aligned objects	0.07	0.26	0.42	1.00	1.34	8.65

them. For example, children might pretend to bake cupcakes by placing the wiffle balls in the muffin tins or treat the pillbox as a house and the pompoms as people who lived in various sections. In contrast, objects-with-objects parents reported that children played with each set of toys one at a time, without trying to combine them (e.g., frogs only or balls only). In cases where the two sets of toys were used together, the role of one-to-one correspondence was often ambiguous. For example, one parent reported that her son set up the eggs as airplane-washing stations that were visited by the airplanes. However, it was unclear whether each plane was assigned to a specific station (i.e., egg).

In summary, children in both conditions played with the experimental toys an equal amount. However, objects-with-slots toys encouraged more one-to-one play, and this appeared to increase children's ability to recognize numerical equivalence in a challenging number-matching task.

DISCUSSION

The ability to group numerically equivalent sets is a fundamental component of emerging numeracy. Numerical similarity is based on one-to-one correspondence—whether this correspondence is established directly between individual items by pairing or indirectly by enumerating one set and then the other. Thus, it seemed likely that experience with one-to-one pairings would lead to better recognition of numerical equivalence. Previous research revealed that preschool children engage in many one-to-one play activities that might provide such experience (Mix, 2002). Of these, placing objects into holes or slots seemed to offer the greatest opportunity for conceptual growth because doing so results in a stable representation of equality that is easily inspected or modified. To see whether this was the case, we provided toys that invited objects with slots play and then tested children's ability to perform a challenging number-matching task. As predicted, children who played objects-with-slots toys improved significantly on the number cross-mapping task following the training period. This effect was quite dramatic given that children had the toys for only 6 weeks and were not explicitly trained in any way. In light of this, it seems reasonable to infer that the sustained experiences with one-to-one correspondence evident in children's spontaneous play (Mix, 2002) make a significant contribution to the growth of equivalence concepts.

By what mechanism might one-to-one pairing lead to an abstraction of numerical equivalence? Research suggests that the comparison process itself is a likely mechanism for recognizing similarity in general (Christie & Gentner, 2010; Gentner, 2003; Gentner et al., 2003; Gentner & Namy,

1999; Namy & Gentner, 2002; Smith, 1989, 1993; Waxman & Klibanoff, 2000). On this view, when children begin to compare two entities for whatever reason, they engage an alignment process that reveals new dimensions of similarity. Thus, any context that encourages comparisons between sets is likely to help children discover numerical equivalence.

Several factors increase comparison between objects, including shared surface features, verbal labels, and thematic relations. Previous research on numerical equivalence indicates these factors also increase comparisons between sets (e.g., Mix, 1999a, 1999b, 2008a, 2008b, Mix et al., 1996; Piaget, 1941). In the present study, we supplied objects that fit into containers with individual openings, or slots. We reasoned that these matches had a degree of surface similarity because the objects were the same size as the openings and could fit into them. This similarity also relates to physical affordances in the Gibsonian sense and therefore seemed likely to elicit fitting the objects and slots together. Objects with slots have another important property related to numerical equivalence. They encourage a one-to-one mapping between individual items in the sets and preserve this mapping in a form that is easily inspected and manipulated. From this, children could access information about numerical equivalence that is not obvious from other encounters with objects.

Although we used physical fit to elicit one-to-one matching, there are many ways object sets could be related one to one. For example, loose objects might be more readily matched if they represent a thematic relation, such as mommy animals and baby animals, or a functional relation, such as cars and drivers. In theory, any relation that promotes one-to-one correspondence should lead to the same effect reported here. An interesting direction for future research would be to determine whether this is so, or if only certain relations lead to the gains we obtained.

Indeed, having multiple cues for one-to-one correspondence (i.e., multiple points of alignment) may lead to greater gains than any one cue alone. The present study hints that this is the case. Recall that children often framed their play in terms of thematic one-to-one relations (e.g., referring to the wiffle balls as cupcakes). The fact that children spontaneously recast the objects into thematically related sets suggests that they either prefer thematic relations or prefer situations with redundant cues. It also indicates that having a single cue for one-to-one correspondence leads children to seek others. This finding is consistent with general theories of comparison and similarity recognition that predict the more cues there are for one-to-one correspondence, the more likely children are to recognize and analyze them.

Given that 3-year-olds typically have amassed years of one-to-one correspondence experiences and can match numerically equivalent sets under at least some conditions, what are we to make of the training effect revealed

in the present study? Why did a mere 6 weeks of exposure to objects with slots lead to gains that normally take a year or more to reach?

First, although toddlers carry out a variety of one-to-one activities in naturalistic play, objects-with-slots activities are neither the earliest nor most frequent. Instead, very young children tend to engage in social activities, such as distributing objects to people or taking turns, or linguistic activities, such as counting or naming objects in sequence (Mix, 2002, 2009). Because objects-with-slots activities emerge relatively late (around age 3 years) and do not make up the majority of children's one-to-one activities even then, it is unlikely that children in the present study had amassed very much of this specific experience. This may be due to the physical requirements of the objects, children's perceptual-motor development, or both. Unlike handing objects out to people, for example, placing objects into slots requires items that fit together. Children may need a great deal of experience with containment to judge whether specific objects will fit into specific slots and, if they guess incorrectly, may become engaged in the containment problem to the exclusion of whatever numerical information might be available. If the objects are not symmetric (e.g., puzzle pieces that only fit into their slots in one orientation), children will need the spatial and motor skills to pair them—skills that do not emerge until late in the 2nd year (Örnkloo & von Hofsten, 2007; Street, James, Jones, & Smith, *in press*). And even if children find objects that fit together and are easily manipulated, it is unlikely they will find them in multiples. Thus, simply providing toys that facilitated these mappings should increase the rate of objects-with-slots experiences versus naturalistic play, even without explicit training.

Second, as we have argued, objects with slots may be more informative than other one-to-one activities because these pairings result in a lasting correspondence that can be inspected, repeated, and modified. An interesting developmental question is whether the various one-to-one activities build on each other or simply constitute different routes to the same understanding. For example, when children hand objects out to people, they are supported by the local correspondences of one object per hand. This experience may prepare children to view objects with slots as an extension of the distributed objects scheme. In this way, one activity could lead to another. Alternatively, children may not connect the two activities and ultimately derive little beyond establishing local correspondences when they distribute objects. In any case, there is reason to believe that objects-with-slots activities give children unique information about numerical equivalence and, thus, have the potential to cause a radical shift in understanding.

Although we provided numerically equivalent sets, it is possible the same effects would be obtained with unequal sets (e.g., a muffin tin with six openings and a set of five wiffle balls). The same mechanisms of comparison and

one-to-one mapping would be engaged and thus could lead to the same increase in numerical equivalence recognition. In fact, the training effect might be more pronounced because nonequivalent sets would provide a contrast to numerical equivalence, much the same way mixing exemplars and nonexemplars in a grouping task might highlight within-category similarity (e.g., asking children to group horses and dogs rather than dogs alone). Moreover, when individual items are aligned, but there are leftover, unmatched items in one set, it indicates that one set is larger than the other. Thus, including nonequivalent sets also might impart a sense of ordinality.

Another interesting direction for future work would be examining the impact of one-to-one mappings on children's understanding of number names/written numerals. In a series of studies, Siegler and colleagues have found children initially fail to judge the relative magnitude of verbal numbers (Opfer & Siegler, 2007; Ramani & Siegler, 2008; Siegler & Opfer, 2003; Siegler & Ramani, 2008). For example, when asked to place written numerals on a number line from 1 to 10, 3-year-olds tend to bunch the higher numbers close together, rather than spacing them evenly. However, children's number line placements become more accurate after playing a simple board game in which children spin a wheel, land on a written numeral, and move that number of spaces (Ramani & Siegler, 2008; Siegler & Ramani, 2008). Like objects-with-slots toys, this board game emphasizes one-to-one correspondence—in this case, the correspondence among count words, moves, and spaces. Perhaps adding a verbal counting component to the objects-with-slots activities would lead to similar gains. For example, children could be encouraged to count the objects as they place them in slots, or count first one set, then the other, and check for equivalence by matching them. In fact, linking number words to objects-with-slots matches might lead to even greater gains than the board game because there are likely to be fewer one-to-one correspondence errors. Such errors are normally quite frequent when preschool children try to map count words to movements and objects (Fuson, 1988) but may be less so on more spatially constrained tasks like placing objects into slots. Moreover, whereas the equivalence between count words and movements is ephemeral in Siegler's board game, it is lasting and inscriptible when objects are placed in slots.

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APPENDIX

OBJECTS-WITH-SLOTS TOYS¹

- | | |
|----------------------|------------------------|
| A) Plastic eggs (6) | Egg carton (6 slots) |
| B) Wiffle balls (6) | Muffin tin (6 slots) |
| C) Puzzle pieces (8) | Puzzle board (8 slots) |
| D) Pompoms (7) | Pill box (7 slots) |

OBJECTS-WITH-OBJECTS TOYS¹

- | | |
|----------------------|---------------------------|
| A) Plastic eggs (6) | Airplanes (6) |
| B) Wiffle balls (6) | Frogs (6) |
| C) Puzzle pieces (8) | Koosh balls (8) |
| D) Pompoms (7) | Plastic spinning tops (7) |

¹Children received three of the four possible toy sets from their assigned condition.