The Role of Adult and Environmental Input in Children's Math Learning

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Parents are children's first teachers. Many parents take that role very seriously and want to do all they can to encourage their child's success in school and life. Early childhood teachers also know that the early years are important years for learning and want to do their best to support the development of young children. To help support children, parents and teachers provide them with various types of what psychologists call "input," such as talking to them and making sure they have experiences with books and toys. But, the question is, how much does the *type* of input matter?

For this chapter, we interviewed Kelly Mix, a cognitive psychologist and a professor at University of Maryland. Mix started her career as an elementary school teacher and was intrigued by the cognitive processes that take place as children are learning. She decided to pursue a graduate degree so that she could study the specific mechanisms by which children learn mathematics and how teacher and parent input can support this learning. In this chapter we will look at three of her studies that explore types of input that help children acquire mathematical concepts and improve in mathematics skills.

Study 1 - The role of one-to-one play in learning numerical equivalence

"This group has four bears and the other group has four cars. They both have the same number." How do children develop the idea of numerical equivalence—that two sets have the same quantity? Usually, when we think about comparing sets, we are thinking about other attributes of those sets. Color, size, and type of object are all features of the sets that we can compare. How do children learn to compare the *numerical* attribute of sets—that is, see what a set of four bears and a set of four cars have in common?? This was the question that Mix and colleagues investigated in the first study we explore.

Mix's work was influenced by the ideas of Northwestern University psychologist Dedre Gentner, who studies the way that children and adults use comparisons between objects or ideas to develop new ideas—a process that she calls "structure mapping." Structure mapping begins when just one similarity is noticed between the entities being compared. Noticing the one similarity leads the learner to align the objects or ideas, "mapping" the common features from one entity to the other. As more features are mapped, more similarities are sought, and whatever common structures there are between the entities are more likely to emerge. That is, when two entities have a similar structure, or frame, as we identify more commonalities between them, the likelihood of our *seeing* the common structure increases. So, the more similarities that exist between two entities, the more likely it is that people will begin to compare them and may see whatever commonalities they have—such as the fact that two sets have the same quantity of members.

Mix recognized the fact that number, more than most of the other attributes we expect children to focus on, must be psychologically imposed by the child onto her experience of the world. That is, it is harder to see the "threeness" in a pile of toy cars than to recognize the features of the individual cars—their "redness" and their "shiny-ness." But "threeness" as a concept does eventually emerge in children's thinking, and Mix wondered if this structure mapping process would be useful in supporting children's recognition of numerical equivalence. In order to test this, Mix needed to set up a situation that would prompt children to make a comparison—to begin the structural alignment process so that quantity as an attribute might be revealed. In considering the various contexts that could cause children to pair objects across sets, Mix asked whether aligning sets of objects so that there are similarities in their spatial arrangement would lead children to compare them numerically, either by counting each set or using one-to-one correspondence.

There are many situations in everyday play where two sets of objects are spatially paired together and comparisons can be made. For example, when children set the table for snack time, they might pair objects (1 napkin for each chair at the table) or distribute objects (give out 1 cookie to each child). Through careful observations of different types of one-to-one correspondence activities with her son between the ages of 12 and 38 months, Mix noticed that one type of activity *came just before* he developed numerical equivalence. The activity involved placing loose objects into a container with slots, such as putting eggs into an egg carton or balls into a muffin tin. This made her wonder whether putting objects into containers—an activity she called "objects-with-slots"—could be a powerful context for surfacing the idea of numerical equivalence.

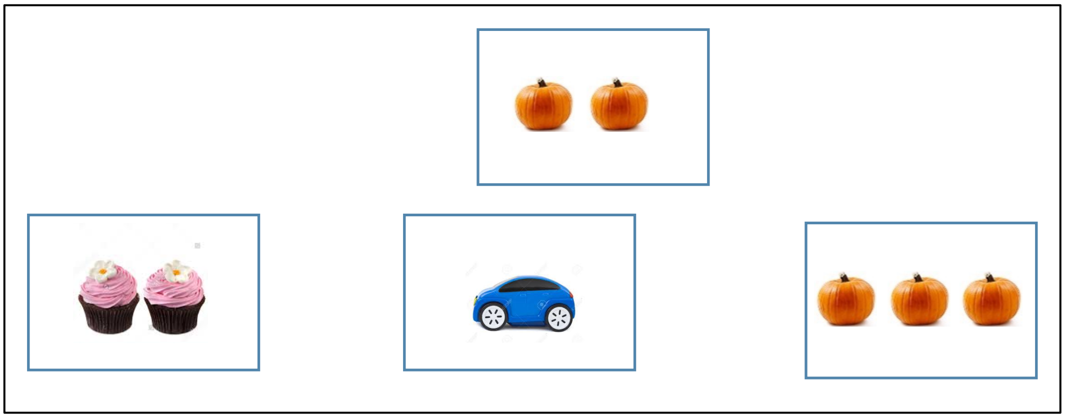
Why might these spatial alignment activities—more than setting the table or handing out cookies—help children to develop an understanding of numerical equivalence? Mix's idea was that "objects-with-slots" activities make the attribute of number visually apparent: the numerical equivalence of the two sets is visible as the slots are being filled. If there is an open slot or there are extra objects to insert, the question of "how many more/less" surfaces very quickly. In other activities, such as distributing objects to people, it is harder to see how the two groups align. In Mix's words, "If children hand out cookies to a playgroup, the sets will be moving. One set will be eating the other--" not a useful context for comparing quantities!

Box 1: One-to-one Correspondence
One-to-one correspondence is a relationship that exists between two sets of objects. When counting, it is the relationship between the set of number names and the set of objects being counted, where one number is named for each object counted. When thinking about two sets of objects, one-to-one correspondence means that for every item in one set, there is a corresponding item in the other set. When there is one-to-one correspondence between 2 sets of objects, there is also numerical equivalence – the two sets have the same number of objects in them.

In this study, Mix wanted to see if exposure to more focused one-to-one correspondence (pairing objects with slots) activities would help children think about how two sets are related numerically. To test this, Mix and her colleagues used a task called the "cross mapping task" that assessed whether children can match sets based only on their quantities.

In the task, children were first shown a target card that had a set of objects on it, and then were asked to select, from a choice of 3 cards, the one that "matched" the target card. Three types of cards were always given as choices. One had the same quantity but different objects than the target card. Another showed the same objects but a different quantity than the target card. A third showed a different quantity and different object than the target card (see Box 2). Before starting the test trials, children were given several training trials in which they were shown that the card that "matched" was the one with the same quantity. This is a difficult task for children because they have to overcome the inclination to select the card with the same object on it. Previous studies had shown that children do not typically succeed on this task until around age 4.

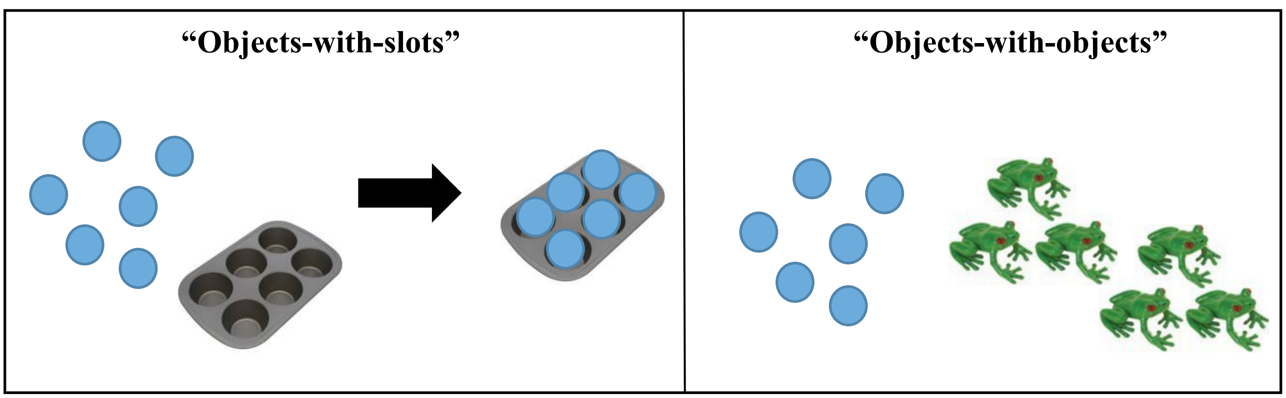
Box 2: Cross-Mapping Task



The researchers tested 37 three year olds with this task. They then excluded from the study the children who performed above chance level (children who got more than 33% correct), since these children might understand numerical equivalence already. This left 30 three year olds.

Next, the researchers divided the children randomly into two groups. Both groups of children were given three sets of toys to take home every two weeks (See Box 3). One group of children was given sets of toys termed “objects-with-slots:” these were toys that included a group of objects and a container with an equivalent number of slots or openings (e.g., 6 whiffle balls and a muffin tin with 6 spaces). The other group was given “objects-with-*objects*” sets of toys. They included two groups of objects that were equivalent in number but had no other connection (e.g., 6 whiffle balls and 6 toy frogs). Parents were told to make the toys available for the children to play with at all times, and they were asked to keep logs that recorded how, and how often, the children played with the sets of toys. They kept each set of toys for two weeks, for a total of 6 weeks of play. At the conclusion of the 6 weeks, the children were tested again on numerical equivalence using the cross-mapping task.

Box 3: Examples of toy sets



The researchers found that while both groups of children performed better on the task after playing with the toys, the group that played with the “objects-with-slots” toys showed greater improvement. In addition, many more of the children in the “objects-with-slots” group performed above chance level (more than 33% of items correct). This suggested that these children were better able to understand numerical equivalence.

The parent logs also revealed interesting information. The children in both groups played with the toys for about the same amount of time, but the difference was in *how* the children played with the toys.

The children in the objects-with-slots group frequently played with both groups of toys, creating play situations where they were putting the objects in and out of the slots (like making cupcakes), while the objects-with-objects group frequently played with the two groups of toys separately. Thus, it seemed that Mix's hypothesis was correct—that the objects-with-slots toys encouraged children to play with the toys such that the two sets aligned.

This study suggests that type of input matters in helping children construct mathematical concepts. Specifically, providing materials that encourage children to make comparisons between sets (objects-with-slots) may help young children construct the understanding of numerical equivalence between sets of objects. Mix argues that the objects-with-slots toys "ground the idea of number and numerical equivalence for them in a physical model."

Study 2 – Acquisition of the cardinal word principle: The role of input

In the next study, Mix and her colleagues investigated how certain kinds of input might help children learn another important mathematical concept: the cardinal word principle. This principle states that the last word in a count represents the quantity of the set (see Box 4). In other words, when counting a set of objects, the last number word you say tells you how many objects are in the set. In this way, learning to count means learning to think about number words in two ways simultaneously, both as markers in a count sequence and as words that describe a total quantity. So how do children discover that the last word in a count stands for its cardinality?

Much research suggests that children must experience an overlap between counting and cardinality to signal that the two ideas are related. Small sets may be ideal for this because children can recognize and name the quantity of small sets without counting (by subitizing, or automatically "seeing" how many there are). If they can count the small set ("one, two, three") and they also know the cardinality (three) by subitizing, perhaps the overlap in number words is enough of a connection to link counting and cardinality.

Box 4: Learning the Cardinal Principle
In order to learn the cardinal principle, children must have several skills. They need to know the counting sequence in the correct order. They also have to be able to coordinate saying the number words and counting the items with one-to-one correspondence, saying one number name for each item touched (tagging). But even when a child knows the number words in order and can count with one-to-one correspondence, they may not know the cardinal principle. For example, if you ask children, "How many blocks do you have?," they may be able to accurately count and tag saying, "one, two, three, four." However if you ask again, "How many blocks do you have?" they may start counting again, "one, two, three, four"—they may not be able to tell you that they have "four." This is an indication that they do not know the cardinal principle. Such children still need to realize that the last number word said when counting tells how many there are in the set.

In this study, Mix again wondered if structure mapping would help children acquire the cardinal principle. In structure mapping, detecting similarities between two groups allows them to be compared and aligned. As more comparisons are made, less obvious relationships may surface. So, if children notice that there are shared number words used in counting a set and in naming the cardinality of the set, they might wonder how else these sets (set of number words and set of objects being counted) are the same. Mix wondered whether certain kinds of input would better support children to see the *similarity* in the word but the *difference* in its use to make the connection between counting and cardinality--in other words, whether children would understand that "three" can represent the quantity of fingers on one hand *and* the word that comes between "two" and "four" when counting.

Mix investigated this question with two experiments. In the first experiment, children received training once a week for 6 weeks involving picture books that displayed sets of objects (e.g. 3 crackers or 4 ducks). Sixty 3½ year olds participated in the study. Children were randomly assigned to one of five conditions that either aligned or did not align cardinality and counting, as described in Table 1. That is, sometimes they heard number words used to count but not to describe a total amount, sometimes they heard number words used to describe a total amount but not to count, and other times they heard both uses, with both directed toward the same set of objects.

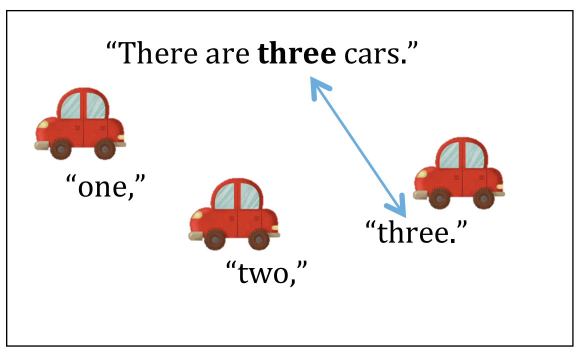
Table 1. Training Conditions

Condition	Counting	Labeling quantity (cardinality) of the set	Example
Comparison	✓	✓	"Look, this page has 3 crackers. Can you say it with me? Three crackers. Let's count them, 1, 2, 3!"
Counting	✓	✗	Look this page has crackers. Let's count them together. '1 (pointing to object), 2 (pointing to object), 3 (pointing to object)'."
Naming	✗	✓	"Look: this page has 3 crackers. Can you say it with me? Three crackers."
Alternating	Alternated across sessions		This group alternated the type of training session – counting the sets one week and then naming the sets the next week.
Control	✗	✗	"This page has crackers. They are yummy. Can you say it with me? Yummy."

To test children's learning, researchers tested them three times: before the first training session, immediately following the third training session, and immediately following the sixth training session. In each testing period, they tested children on various counting skills, including one intended to measure their understanding of the cardinal principle (asking children to produce, from a pile, a certain number of objects).

The researchers found that only children in the Comparison condition improved on these tasks. In fact, after three weeks of training in the Comparison condition, children demonstrated an understanding of the cardinal word principle, but even 6 weeks of training with the other conditions was not sufficient. This study suggested that labeling the cardinality and then immediately counting a variety of sets helped children to connect counting and cardinality, whereas providing the same amounts of counting or labeling alone did not help. In other words, providing a structure that helped children map the cardinal label to the last number word counted helped children learn the cardinal principle.

Box 5: Input that provides both the cardinal label and counting



Next, Mix and her colleagues asked what kind of input parents typically give to children regarding number. Parents were videotaped as they read two trade books to their preschool children – one that was about number and one that was not. Parent language was then coded to determine how often they specifically provided input that included both the cardinal label as well as counting. Results of this experiment indicated that the parents almost always read the text of the book, regardless of the book's content. When parents elaborated on the book, they commented on non-numerical information much more frequently than they did on numerical information. Even when they did elaborate on the numerical information, the number of times that they labeled the quantity of the set and then counted the set was small (only once out of 79 utterances), even when using a book about number.

Typically, it takes children about 18 months for acquisition of the cardinal word principle to occur, if you start measuring from the time they start learning the counting sequence. This study showed that after experiencing labeling and counting of the same sets in close time proximity, there was a rapid development of cardinal understanding. "Label + count" training worked after only 3 sessions. The findings seem to indicate that children can acquire the cardinal word principle rapidly once the right type of input is provided. At the same time, the typical input provided by parents is not input that is going to support a deeper understanding of the cardinal word principle.

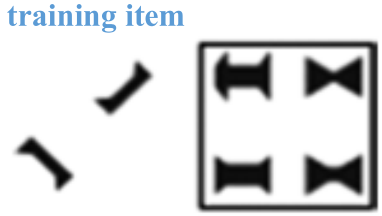
Study 3 – The effect of spatial training on children's mathematical ability

In the two studies above, Mix examined the ways that children create connections between mathematical concepts and what kinds of input might support those connections. In her more recent work, as she told us, she takes a step back to think about what mechanisms children use to make these sorts of connections, and whether at least some of these are spatial in nature. If they are, it is possible that training on spatial thinking could improve math performance.

Spatial thinking involves perceiving objects in space, thinking about how they relate to one another and to the viewer, and visualizing how they might look when turned or moved. Many studies have demonstrated that people who are good at spatial tasks also do well in mathematics. This may be because they are better able to use and reason about spatial representations of numerical ideas such as number lines and graphs. However, Mix and her colleague Yi-Ling Cheng wrote a thorough literature review on the connection between space and math and found a firm basis to conclude that spatial ability and math share cognitive processes that begin early in development. Their next question was whether spatial training can improve children's mathematical ability.

Fifty-eight 6- to 8-year-old children participated in the study. First they were given a math pre-test. This included single-digit number fact problems (e.g., 4+5=__), two and three-digit calculation problems (56-6=__; 124+224=__), and missing term problems (4+__=12). There were both addition and subtraction problems within the three problem types.

Box 6: Sample mental rotation training item



Next, children were randomly divided into 2 groups: a spatial training group and a control group. The spatial training involved a 40-minute session focused on mental rotation. Children saw two parts of a flat shape and then were asked to find the shape, out of four choices, that showed what the parts might look like when they were rotated and combined (see Box 6). After making their selection, the children were given the two parts on separate pieces of cardstock, and they were able put the two pieces together, either confirming their choice or indicating what the correct choice is. Children in the control group worked on crossword puzzles during the 40-minute session. After the training session, they completed the same test that they did before the training.

The researchers found that the children in the spatial training group improved significantly more than the control group on the math test. Why would this be? To investigate further, the researchers looked at performance on the 3 types of problems separately. They found that the spatial training group did significantly better on the missing-term problems (4 +__=12). The researchers did not have strong ideas about how the training helped the children. One suggestion was that perhaps the training helped the children to solve the problems by mentally rotating them to a more conventional form (e.g. 4 +__=12 becomes 12 - 4 =__).

Mix is still exploring the underlying processes that connect spatial ability and other non-spatial math skills. She recognizes that more needs to be done to understand the underlying processes that children engage in and the connection between spatial training and mathematical ability and how that might be leveraged by educators. However, this study was the first to show a direct effect of spatial training on math performance, and that certain types of problems are impacted more than others. Mix says, “the work is still emerging and somewhat controversial, but I think it’s a direction worth pursuing.”

Conclusion

Kelly Mix has been intrigued by the cognitive processes that take place as children engage in learning. Much of her work focuses on how input – whether adult or environmental - impacts children’s thinking about foundational math concepts. There are visual and structural components in all of the studies we described. All of this research points to the importance of making connections – both between mathematical ideas and the world around us, and between different ways of representing mathematical ideas. Parents and teachers can support children in making these connections by providing enough exposure for children to have time to make the connections, through making modifications to the input that highlights the connections, and through general spatial thinking.

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